APPROXIMATE METHOD FOR CALCULATING CURRENT PULSATIONS CAUSED BY INDUCTION MOTORS DRIVING RECIPROCATING COMPRESSORS

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Abstract:

Induction motors driving reciprocating compressors experience fluctuating electromagnetic torques at the rotor, which in turn cause current pulsations in the stator windings. We present an approximate method for calculating these current pulsations based on estimates of the fluctuating electromagnetic torques, and on basic electrical information about the motor that is readily available early in a design project. The electromagnetic torque fluctuation estimates are derived using a method developed for torsional vibration analysis presented at GMRC 2013, and further confirmed by field measurements. The present work of predicting current pulsations is intended to give the designer an approximate, but conservative, method to evaluate current pulsations against the 40% limit for induction motors specified in API 618.

Introduction:

The steady-state torque and power output of a 3-phase induction motor are the result of electromagnetic fields which act across the air gap between stator and rotor. If the rotor has a torsional vibration (speed fluctuation) superimposed over the steady rotation, the same electromagnetic fields across the air gap produce additional mechanical and electrical effects on both rotor and stator respectively. On the rotor, there are additional torques which act as torsional springs or dampers; on the stator the angular speed variation alternately generates and removes energy, producing current fluctuations in the stator windings.

Analytical methods for evaluating these additional electromagnetic (em) spring and damper effects on the rotor were presented at GMRC 2013 [1]. Adding the em spring and damper to standard torsional vibration models resulted in significant shifts in predicted torsional natural frequencies in some cases; particularly so for systems with soft couplings (e.g. rubber-in-shear). Comparing field data for several cases indicated good agreement between the predicted and measured torsional natural frequencies.

The same torsional vibration amplitudes at the rotor also produce positive current pulsations in the stator windings as the rotor momentarily turns faster than the steady rotational speed, thereby acting as a generator. Negative current pulsations occur when the rotor is momentarily turning slower than the

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steady rotational speed. This effect is well-known and in the past several methods for calculating the current pulsations have been widely used (principally by motor manufacturers). These methods are based on first estimating the fluctuating torque produced by a reciprocating compressor, and then using an approximate equivalent electrical circuit description of the motor to infer stator current pulsations.

We have extended our previous *em* spring-damper model [1] in order to calculate the fluctuating *em* torque and thereby provide an estimate of the current pulsations at an early stage in the torsional design of a motor driven reciprocating compressor system. First, we present a brief review of the *em* spring-damper model and its effect on torsional vibration analysis; then an extension to current pulsation estimates follows, based on fluctuating torques calculated from the *em* spring-damper model.

1. Electromagnetic effects on torsional vibration analysis:

Unsteady *em* effects have been extensively studied in the past by direct numerical integration of the non-linear differential equations representing stator and rotor currents and their mutually induced stator and rotor fields. Jordan *et al* [2, 3], have developed the complete *em* field equations, which were then solved numerically showing dynamical effects such as limit cycles and drive instabilities (“negative” damping). More recently, Knop [4] presented a simplified approach to solving the *em* field equations by a “linearization” technique. Using the model suggested by Knop [4] and further estimates made from Jordan *et al* [2, 3], we developed formulas for estimating the *em* spring and damper values to be used in a torsional vibration model as shown in Figure 1.

Figure 1. A representative torsional mass-elastic model of a motor driven reciprocating compressor drive train, with an additional *em* spring and damper acting on the rotor (taken from [1]).
As outlined in our GMRC-2013 presentation [1], values for the *em spring, $k_M$, and* *em damper, $d_M$, can be estimated from:

$$k_M = (\# \text{ stator poles})(T_B \{x^2 / [1 + x^2]\})...\text{Eq'n (1)},$$

and

$$d_M = k_M / (\omega^2 T_L) = k_M(T_L) / (x)^2...\text{Eq'n (2)},$$

where;

- $T_R$ = rated motor torque, [Nm]
- $T_B$ = breakdown torque. [Nm]
- $s_R$ = slip at rated load, [%]
- $\Omega_s$ = supply frequency, [rad/s]
- $\omega$ = torsional vibration freq., [rad/s]
- $T_L$ = electrical time constant, [s],
- $x = (\omega T_L)$.

The electrical time constant of the motor $T_L$ can be estimated from:

$$T_L \approx (1/\Omega_s)[1/(2s_R)]\{T_R / T_B\}...\text{Eq'n (3)}.$$

While the main motor torque characteristics $T_B$, $T_R$ and $s_R$ are either tabulated or readily available, the time constant $T_L$ requires a further evaluation of Eq'n (3). Figure 2 shows estimates made using Eq'n (3) for two types of motors, with time constants $T_L$ ranging from 0.07 to 0.14 seconds in the range of 500 to 1,500 kW.

![Figure 2](image_url)

*Figure 2*. Estimated time constant $T_L$ for typical induction motors; data taken from published information [5, 6]. The dashed line is for reference only (taken from [1]).
The addition of the \textit{em} spring, $k_M$, shown in Figure 1 creates an additional vibration mode, with the original lowest torsional natural frequency (TNF) split into both a lower and upwardly shifted value as Knop [4] explained. Figure 3 shows the predicted upward shift for three representative torsional drive trains with different stiffness couplings, $k_C$: Case A, with a torsionally soft coupling (rubber-in-shear); Case B, with a steel spring coupling; Case C, with a stiff coupling (disc pack).

![Graph showing TNF shift](image)

**Figure 3.** Representative \textit{em} effects for drive trains having couplings with different stiffness (taken from [1]). The inertia ratio lines are lumped inertia totals for the motor side, $J_M$, and compressor side, $J_C$ (shown for reference only). The three values shown for Case A are due to the variation of rubber coupling stiffness with temperature during operation.

Figure 3 shows that with stiff couplings ($k_C \gg k_M$) the TNF is not significantly affected, while with soft rubber-in-shear couplings ($k_C << k_M$) the higher TNF can be raised into the operating speed range [4]. As a result it was suggested [1] the \textit{em} effects should be included in all torsional vibration analyses (TVA) of drive trains with soft rubber couplings.

An \textit{em} spring and damper with $k_M$ values estimated using Eq’ns (1), (2), and (3) were added to a series of previous TVAs where field measurements of the TNF were available. Figure 4 shows the measured TNF (RPM) where the \textit{em} was naturally present, versus the re-calculated TNF with \textit{em} effects included in the TVA. (solid markers). The proximity to the 45° line indicates good agreement between the \textit{em} model and field measurements, particularly so for soft rubber couplings where the TNF shift can be significant.
Figure 4. Comparison of predicted versus measured TNF when *em* effects are included in the TVA (solid markers), and when they are not included in the TVA (open markers). This Figure shows that including the *em* effects in TNF estimates is particularly important for torsionally soft (rubber-in-shear) couplings.

We continue to seek field measurements for further confirmation of the *em* effects and the usefulness of Eq’ns (1) and (2). We suggest the *em* spring and damper values based on Eq’ns (1) and (2) presented here provide reliable estimates of the *em* effect and their inclusion is recommended for TVA studies.

2. Estimating current pulsations arising from torsional vibration:

(a) Estimating rotor torque pulsations based on electromagnetic effects;

Estimating rotor torque pulsations is the first step in estimating current pulsations. If a TVA has been carried out including *em* effects, the torque pulsations on the rotor are directly available. If *em* effects have not been included in the TVA, the previously described methods [1] for estimating the *em* spring and damper values acting on the rotor can be combined and extended to find the total fluctuating torque across the motor air gap (*mag*). A direct method following suggestions by Knop [4], and the results from our previous work [1] is outlined in the Appendix. The resulting estimate for the fluctuating *mag* torque becomes

\[ \Delta \tau = \frac{(T_{\text{MAX}} - T_{\text{MIN}})}{2T_R} = \left(\text{# stator poles}\right)\left(\frac{T_B}{T_R}\right)\left(\theta_0\right)x \left(1 + x^2\right)^{1/2} \] …Eq’n (4),
where $\Delta\tau$ is the relative fluctuating mag torque, Eq'n (4), and $\theta_o$ is the torsional vibration amplitude (0-pk) at the rotor (all other terms as previously defined). Rotor amplitudes are largest for drives with stiff couplings (disc pack), and a preliminary TVA without em effects can be used to evaluate $\theta_o$ for use in Eq’n (4). Torsional vibration amplitudes at the rotor can also be estimated from strain gauge measurements on the motor output shaft (assuming a rigid rotor model).

(b) Estimating current pulsations based on torque pulsations;

Torsional vibration and attendant torque pulsations at the rotor have long been known to be capable of producing strong stator current pulsations. Guidelines for the designer in API 618 suggest the current pulsations should be less than +/- 40% of the primary motor current, and suitable modification to the drive train (principally modifying the flywheel inertia) is required to stay within this guideline.

Approximate methods for calculating the stator current pulsation based on estimates of vibratory mag torque pulsations at the rotor have been used (mostly by motor manufacturers) for some time [7, 8, 9]. Concordia [7] developed a method for estimating current pulsations for induction motors using the concept of “synchronizing” and “damping” torques, later extended to synchronous motors by Halberg [8]. A detailed analysis and computer program (RECIP) was developed by Cummings [9], which accounts for the mag torque pulsations on an individual harmonic basis. Cummings also reported that direct measurement of the stator current compared very closely to the predicted current. Cummings further suggested that if only the magnitude of current pulsations was required, the current pulsations could be estimated from the vector diagram in Figure 5.

![Figure 5. Vector diagram taken from Figure 5 of Cummings [9], showing the construction for estimating the magnitude of the total maximum current (reported as being “utilized by the author’s company since the late 1950’s”). Nomenclature: Y, peak-to-peak mag torque pulsation; I, primary current; current subscripts R, Q, M are the primary current direct, quadrature, and magnetizing components.](image-url)
We have redrawn Figure 5 as shown in Figures 6 (a), and (b) in order to provide a means for making current pulsation estimates based on readily available motor data. The primary current is shown having unit value, so that the direct and quadrature components are both determined simply by knowing the rated power factor, \( \text{PF} = \cos \phi \).

![Vector constructions](image)

**Figure 6 (a), (b).** Vector constructions suggested by Cummings [9], showing the methods for evaluating both maximum (a) and minimum (b) current, relative to a primary current of unity. The diagram has been redrawn for a typical large motor-compressor installation; \( \text{PF} = 0.8, \Delta \tau = 0.3, \) and \( m = 0.3 \) (not to scale).

Referring to Figure 6 (a), the maximum current (for unity primary current) can be expressed as

\[
I_{\text{MAX}} = (\cos \phi)(1 + \Delta \tau)\{1 + [\tan \phi - (m/\cos \phi)(\Delta \tau)/(1 + \Delta \tau)]^2\}^{1/2} \quad \text{Eq’n (5),}
\]
where \( m \) is the ratio of magnetizing current to primary current, and the torque pulsation, \( \Delta \tau \), \((0\text{-pk})\) is for a primary current of unity. In many cases of interest Eq’n 4 can be simplified since \((\tan\varphi)^{-1} > (m/cos\varphi)(\Delta \tau)/(1+\Delta \tau)\), so that as an approximation

\[
I_{\text{MAX}} \approx (1+\Delta \tau)[1-(m\sin\varphi)(\Delta \tau)/(1+\Delta \tau)] \quad \text{Eq’n (6)}.
\]

The expression for the minimum current is

\[
I_{\text{MIN}} = (\cos\varphi)(1-\Delta \tau)[1+(\tan\varphi + (m/cos\varphi)(\Delta \tau)/(1-\Delta \tau)]^{1/2} \quad \text{Eq’n (7)},
\]

which with the approximation cited above becomes

\[
I_{\text{MIN}} \approx (1-\Delta \tau)[1+(m\sin\varphi)(\Delta \tau)/(1-\Delta \tau)] \quad \text{Eq’n (8)}.
\]

Using Eq’ns (6) and (8) for the case described in Figure 6 (a) and (b), the estimate for \( I_{\text{MAX}} \) made with Eq’n (6) is about 0.5% low, while the result for \( I_{\text{MIN}} \) with Eq’n (8) is about 0.8% high. Subtracting Eq’n (8) from Eq’n (6) gives a simple but useful result for the relative current pulsations as

\[
(I_{\text{MAX}} - I_{\text{MIN}})/2 = \Delta I \approx \Delta \tau(1-m\sin\varphi) \quad \text{Eq’n (9)},
\]

where \( \Delta I \) is the current pulsation \((0\text{-pk})\) relative to a unity primary current.

The \textit{mag} torque pulsation \( \Delta \tau \) in Eq’n (9) can be estimated from Eq’n (4) using the results of the \textit{em} spring and damper values of the previous section. The power factor, \( PF = \cos\varphi \), is usually stated for any motor load, however the magnetizing current fraction, \( m \), may be more difficult to obtain. If the \textit{no load} current is known, this approximately equals the magnetizing current. Since the magnetizing current stays essentially constant as the load increases, the value of \( m \) for other loads is found directly. As a rough guide, most large induction motors running at full load have \( m \approx 0.3 \), while at half load \( m \approx 0.5 \).

3. \textbf{Case Histories. Comparing different methods of predicting current pulsations with measured current pulsations:}

Many years ago, one of the authors (BCH) raised questions about the supplier-predicted current pulsations during a design project. The motor manufacturer insisted that the current pulsation would be excessive unless a very large flywheel was installed (so large in fact, that a steady bearing had to be added to support the weight!). Although the suggested current pulsation was unbelievably high (based on experience with similar sized units), the motor manufacturer threatened a loss of warranty unless their recommendations were followed, He (BCH) was unable to verify the current pulsation prediction, and thus was not able to help the end user avoid the extra expense of the required (but
unnecessary) modifications. The method outlined above is intended to help others that may find themselves in the same situation.

The Case Histories that follow compare various past methods for estimating current pulsation, actual field measurements where available, and the estimates made using the em methods described above. The results are summarized in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Torque pulsations</th>
<th>Current pulsations</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 Cummins</td>
<td>122%</td>
<td>78%</td>
</tr>
<tr>
<td>#2 TECO</td>
<td>15.3%</td>
<td>11.0%</td>
</tr>
<tr>
<td>#3 Hyundai</td>
<td>8.9%</td>
<td>7.2%</td>
</tr>
<tr>
<td>#4 Baldor</td>
<td>8.9%</td>
<td>7.0%</td>
</tr>
</tbody>
</table>

Footnote: Torque and current pulsations in Eq’ns (4) and (9) are multiplied by 2 and shown as % for ready comparison with the NEMA measurement method and API 618 requirements.

Table 1. Comparison of results using different methods of predicting current pulsations, and also comparison with measured results where available.

4. Discussion of results:

Case History #3 involves a motor driving a 6 throw compressor which experienced compressor crankshaft and motor rotor failures. Improvements to the torsional design of the system resulted in acceptable operation of the unit. Current pulsations were measured and found to be in the predicted range. The “current-speed slope” method provides an estimate of 200%.

In Case History #4 current pulsations were measured at the site. The measured current pulsations showed good agreement with those predicted by Eq’n (9), first using TVA results to estimate the torsional amplitude in Eq’n (4) and so obtain the torque pulsation. There was some disagreement between the predicted and measured speed fluctuations; at this time it appears that the method of measuring rotor fluctuating speed is subject to error due to runout of the quill shaft, so the 1x contribution was removed. The “current-speed slope” method provides an estimate of 106 % (using the NEMA method of calculating current pulsations).
It may be seen from Table 1 and Eq'n (9), that current pulsations are always less than torque pulsations (on a per unit % basis). Using the torque pulsation estimates from Eq'n (4) provides a most conservative estimate of current pulsations for meeting the requirements of API 618.

5. Conclusions:

1. A new method of predicting current pulsations is presented that permits induction motor users and torsional analysts to predict current pulsations with minimal and easily obtained information.

2. At the outset, we note that current pulsations must always be less than the torque pulsations on a per unit basis as can be seen from Eq'n (9). A deliberately conservative but useful estimate of current pulsations can first be made from the torque pulsation estimate, Eq'n (4), and assuming that current and torque pulsations are equal.

3. The new predictions are consistent with field measurements and with other more complicated prediction methods.

4. This new method provides a more rational approach to estimating current pulsations than the often used “mean current-speed slope” method. The “mean current-speed slope” method should not be used to estimate current pulsations as it much too inaccurate. Its use has resulted in unfortunate compressor designs with huge flywheels.

5. Agreement between measured and predicted current pulsation using this new method should be judged relative to the acceptance criterion in API 618. Error in the predictions that the authors have been able to check are consistently small relative to the 40% current pulsations allowed for induction motors by API 618.
REFERENCES:


APPENDIX:

Knop [4] applied linearization techniques to derive his Eq’n (3), and provide a simplified estimate of the fluctuating motor air gap torque, \( m_M \), as

\[
  m_M = \left[ \frac{M_{st}}{1 + j \omega T_L} \right] \cdot [s],
\]

where \( M_{st} \) is a constant with dimensions of torque, s is the motor slip, \( \omega T_L \) is as previously defined, and the terms in the square brackets are taken as phasors.

Rewriting the above in the notation used here and in [1], the fluctuating torque becomes

\[
  T = \{M_{st}/(1+ x^2)\} \{ 1 - jx \} \cdot [s].
\]

The slip phasor can further be approximated by \([s] = [ j (\theta_o \omega/\Omega_s)]\), where \( \theta_o \) is the +/- angular amplitude of torsional vibration at the rotor. The torque expression becomes

\[
  T = \{M_{st}/\Omega_s T_L/(1+ x^2)\} \{ \theta_o x \} [ x + j ],
\]

with the further result from [1],

\[
  T = \{(\text{# poles})T_B/(1+ x^2)\} \{ \theta_o x \} [ x + j ].
\]

The phasor \( [ x + j ] \) has magnitude \( (1 + x^2)^{1/2} \), so that the relative torque pulsation becomes

\[
  \Delta \tau = (T_{\text{MAX}} - T_{\text{MIN}})/2T_R = \{(\text{# stator poles})( T_B / T_R)( \theta_o )[ x / (1 + x^2)^{1/2} ]\} \ldots \text{Eq’n (4)}.
\]