CMVA – Vibration Math

This formula sheet will be provided with the CMVA exam for Cat 2 and 3 but there is no guarantee that the exam will have related questions. Instead, make sure you master the stated performance objectives for the category you are writing, as well as for the lower categories.

General Note: All the math formulae are written in the standard mathematical way which is independent of units as long as consistency is maintained. All amplitudes are therefore zero to peak, including displacement. When applying these formulae with English units, use inches for length, pounds-force (lbf) for force and seconds (sec) for time, with "g" in ins/sec/sec. (ins/sec^2).
For metric units, although S.I. strictly applied uses metres (m) for length, for this subject millimetres (mm) are the convention and are used here in most places. Newtons (N) are the metric units for force, kilograms (kg) for mass and seconds (s) for time with "g" in mm/s/s (mm/s^2).
Mils, feet, microns (μm), and peak to peak unit conventions all have to be converted to the standard units and conventions before doing any parameter conversions (e.g. displacement to velocity) and the reverse is true when you want your answer in these units. See Page 3.

Frequency of vibration, f (Hz).

$$f = \frac{1}{T} \text{ Hz}$$

where T = period of vibration in seconds.
Hz = Hertz = cycles per second = CPS = cps.

Motion - Basic conversion formulae
Velocity from Displacement:

$$V = 2\pi f D$$

where V is peak velocity,
D is **peak** displacement,
f is frequency in Hz.

Acceleration from Velocity:

$$A = 2\pi f V$$

where A is peak acceleration ,
V is peak velocity and
f is frequency in Hz.

English units.
acceleration: inches per second per second, written ins/sec/sec or ins/sec^2.
velocity: inches per second, written ins/sec or IPS.
displacement: inches, written ins.

Metric units.
acceleration: millimetres per second per second, written mm/s/s or mm/s^2.
velocity: millimetres per second, written mm/s.
displacement: millimetres, written mm.

Acceleration from Displacement:

$$A = \left[2\pi f\right]^2 D$$

where D is **peak** displacement .
English units for displacement: inches (ins) .
Metric units for displacement: millimetres (mm).

Convert acceleration from standard units to multiples of the acceleration due to gravity (g-s):

$$A(\text{units of g}) = \frac{A \text{ in dimensional units}}{g}$$

English units:
A is in ins/sec^2,
 g = 386.1 ins/sec^2 .
Metric units:
A is in mm/s^2,
 g = 9810 mm/s^2.

Conversion of displacement, velocity and acceleration by manipulating formulae

$$V = 2\pi f D \qquad D = \frac{V}{2\pi f}$$

$$A = 2\pi f V \qquad V = \frac{A}{2\pi f}$$

$$A = \left[2\pi f\right]^2 D \qquad D = \frac{A}{\left[2\pi f\right]^2}$$

Spring Force

$$F_x = K_x x$$

where F_x is the force in a spring in the x direction produced by deflection x (from undeformed condition) and K_x is the stiffness of the spring in the x direction.

English units
Force: pounds force (lbf).
Spring stiffness: pounds force / in (lbf / in).

Metric units
Force: Newtons (N).
Spring stiffness: Newtons / mm (N/mm) or Newtons / m (N/m).

Damping Force

$$F_d = C v_r$$

C = Damping coefficient (force per unit velocity):
v_r = relative velocity.
English units.
Force: pounds force (lbf).
C is in pounds force - seconds per inch (lbf sec/in).
v_r is in ins/sec.
Metric units.
Force: Newtons (N).
C is in Newton - seconds per millimetre (Ns/mm) .
v_r is in mm/s.

Data Acquisition Time T for a single continuous sample (secs).

$$T = \frac{N}{F_{max}}$$

where N is number of spectral lines and F_{max} = frequency span in Hz.

Total Data Acquisition Time T_{total} for multiple sample average with overlap.

$$T_{total} = \frac{N}{F_{max}} \left[1 + (n_s - 1) \times \left(1 - \frac{P\%}{100} \right) \right]$$

where n_s = number of samples in average,
P% is percentage overlap.

Lowest Resolvable Frequency LRF (also called Band Width but not recommended).

$$LRF = \frac{F_{max}}{N}$$

Note the term Band-Width has been widely adopted by other disciplines with a quite different meaning.

Resolution

In order to resolve two adjacent frequency components in most instances with Hanning, uniform (rectangular) or flat-top windows, the following formula will give the measurement resolution for most situations:

$$\text{Effective Resolution (Hz)} = 2 \times \frac{F_{max}}{N} \times WF$$

where WF = Window Factor.
WF = 1.5 for the Hanning window, 3.8 for the flat top window and 1.0 for the Uniform (rectangular) window (Refs. 1, 2).

Amplitude in deciBels (dB).

$$\text{Amp (dB)} = 20 \, \text{Log} \left[\frac{V}{V_r} \right]$$

V = voltage (or other measure)
V_r = reference value of voltage or other measure having same units as numerator.

Relative Amplitude in deciBels V_2 , V_1.

$$20 \, \text{Log} \left[\frac{V_2}{V_r} \right] - 20 \, \text{Log} \left[\frac{V_1}{V_r} \right] = 20 \, \text{Log} \left[\frac{V_2}{V_1} \right] dB$$

Dynamic Range

Dynamic range is the difference between the highest (V_H) and lowest (V_L) level signals measurable in terms of the voltage ratio, usually expressed in deciBels (dB).

$$\text{Dynamic Range} = 20 \, \text{Log} \left[\frac{V_H}{V_L} \right] dB$$

Undamped Natural Frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

f_n = undamped natural frequency in Hz of a single degree of freedom system or of a principal mode of a system.

English units:

K = stiffness, lbf/in,

M = mass = w /g ; w = weight in lbf.

g = gravitational constant 386.1 in/sec^2.

Metric units

k = stiffness in Newtons/metre (N/m).

M = mass in kg.

Machine Calculations

Magnetic frequency (F_m) = 2 *times* line frequency *divided by* number of poles (or line frequency divided by number of pairs of poles).

Induction Motor Slip frequency (F_s) = F_m - F_r where F_r = rotational speed of rotor.

Gearboxes (simple speed increaser or reducer).

Gearmesh frequency (GM) is given by:

$$GM = F_1 N_1 = F_2 N_2$$

where:

Input speed is F_1, input number of teeth is N_1.

Output speed is F_2, output number of teeth is N_2.

$$F_2 = F_1 \times \frac{N_1}{N_2}$$

Bearing Calculations (Hz)

Fundamental train frequency (cage frequency):

$$FTF = \left(\frac{1}{2}\right)\left(\frac{RPM}{60}\right)\left(1 - \frac{B\cos\Phi}{P}\right)$$

Inner race ball pass frequency:

$$BPFI = \left(\frac{n}{2}\right)\left(\frac{RPM}{60}\right)\left(1 + \frac{B\cos\Phi}{P}\right)$$

Outer race ball pass frequency:

$$BPFO = \left(\frac{n}{2}\right)\left(\frac{RPM}{60}\right)\left(1 - \frac{B\cos\Phi}{P}\right)$$

Ball spin frequency:

$$BSF = \left(\frac{P}{2B}\right)\left(\frac{RPM}{60}\right)\left(1 - \left(\frac{B}{P}\right)^2 \cos^2\Phi\right)$$

where:

RPM = machine speed revolutions per minute

Φ = contact angle

P = pitch diameter

n = number of rolling elements

B = ball or roller diameter

Roll rotational speed

The speed (frequency of rotation) f_r of a roll:

$$f_r = \frac{V}{\pi D} \quad Hz$$

where V = web velocity.

D = roll diameter.

Note units must be consistent.

Pulley Speed

$$D_1 f_1 = D_2 f_2$$

where:

D_1 = diameter of drive pulley.

f_1 = speed of drive pulley, rpm or Hz

D_2 = diameter of driven pulley

f_2 = speed of driven pulley, rpm or Hz

Note both speeds have to be in the same units.

Belt Frequency for a 2 pulley system.

$$f_b = \frac{2\times\pi\times R_1 \times RPM_1}{60\times l} = \frac{2\times\pi\times R_2 \times RPM_2}{60\times l} \quad Hz$$

where: f_b is the rotational speed of a drive belt and in the case of a defect on the belt is the frequency of impacts of the defect on each pulley,

R_1 = radius of pulley 1,

R_2 = radius of pulley 2,

RPM_1 = speed of pulley 1 RPM,

RPM_2 = speed of pulley 2 RPM,

l is the belt length.

Force due to Unbalance
The rotating force F due to unbalance is given by:
$$F = Me\omega^2$$
where:

ω = angular velocity (omega) $= \dfrac{2\pi N}{60}$ rads/sec

In English units:
M = rotor mass = W/g.
W = weight of rotor in lbf.
e = rotor eccentricity in inches.
g = gravitational constant, 386.1 in/sec^2.
N = machine speed in rpm.
F is unbalance force in pounds force (lbf).
In Metric units:
M is rotor mass in kg.
e = rotor eccentricity in metres.
N = machine speed in rpm.
F is unbalance force in Newtons.

Conversions
To convert mm into inches divide mm by 25.4.

To convert microns (micrometers μm) to mils
divide by 25.4.

To convert inches into mm multiply inches by 25.4.

To convert mils to microns (micrometers μm)
multiply by 25.4.

Hz = CPM divided by 60 = CPS = cycles/second.

CPM = Hz *times* 60.
where CPM = cycles / minute.

For a pure sine wave, Peak = RMS *times* the square
root of two, RMS = Peak *divided by* the square root
of two, Peak-to-peak = 2 *times* peak.

References:
1. J. Frarey *An Examination of Signal Resolution
   in an FFT Analyzer*, Vibrations Vol. 13 Number
   2 June 1997.
2. CMVA Note by Tony Taylor *Effective
   Resolution vs Lowest Resolvable Frequency.*