Operational modal analysis to identify modal parameters in reciprocating compressors

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Abstract

A classical experimental modal analysis (EMA) is typically used to identify the modal parameters (natural frequency, mode shape, damping ratio) of a mechanical system. An EMA applies a known and measurable excitation force to calculate a set of frequency response functions (FRF) and estimate the modal parameters.

For massive structures, such as foundations, large cylinders or piping, it is impractical to use EMA. The artificial forces generated in a typical EMA are insignificant compared to the mass and size of the mechanical system under study.

For large structures, an operational modal analysis (OMA) can be performed to extract the modal parameters. The main difference to an EMA is that the excitation forces are unknown in the OMA. This means the theoretical framework in the OMA needs to be different from a known (deterministic) input/output relation or frequency response function (FRF). In an OMA, the input is assumed to be a so-called stochastic process. There are different algorithms to determine modal properties from an OMA, i.e., data captured during random excitation. When dealing with rotating machinery, the harmonic excitation of the structure caused by the equipment needs to be removed.

This paper discusses the theory and application of OMA on reciprocating machinery and piping modal analysis, including case studies in which OMA was used to identify modal properties of a compressor package foundation, cylinders and piping modes. In these cases, EMA could not be used to calculate modal properties due to the large mass of the structures.
Introduction

A classical experimental modal analysis (EMA) is typically used to identify the modal parameters (natural frequency, mode shape, damping ratio) of a mechanical system. EMA methods are generally used to determine the dynamic characteristics of small and medium-size structures. In rare occasions, these methods are used on very large structures because of the complexity associated with providing significant levels of excitation to a large, massive structure [Ref. 2].

In EMA, controlled forces are applied to a structure to induce vibrations. By measuring the structure’s response to these known forces, one can determine the structure’s modal parameters. The measured excitation and response time histories are used to compute frequency response functions (FRFs) between a measured point and the point of input. These FRFs can be used to determine the natural frequencies, mode shapes and damping values of the structure using well-established methods of analysis.

For massive structures, such as buildings, bridges, dams, large mechanical equipment, etc, it is impractical to use EMA. The artificial forces generated in EMA are insignificant compared to the mass and size of the massive structures. The engineering field that studies the modal properties of systems under ambient vibrations or normal operating conditions without the need to measure the input excitation is called operational modal analysis (OMA) [Ref. 2].

An OMA can be defined as the modal testing procedure that allows the experimental estimation of the modal parameters of the structure from measurements of the vibration response only. The idea behind OMA is to take advantage of the natural and freely available excitation due to ambient forces and operational loads (wind, traffic, random excitations, etc) to replace the artificial excitation. So, they are not considered as disturbance but, on the contrary, allow the dynamic identification of large structures. Since OMA requires only measurements of the dynamic response of the structure in operational conditions, when it is subjected to the ambient excitation, it is also known under as ambient vibration modal identification or output-only modal analysis.

Figure 1 shows a view of the Millau Viaduct bridge in France. This bridge is the tallest vehicular bridge in the world. It has seven concrete piers that range in height from 250 to 800 feet. The total span of the bridge is 1.5 miles. The only way to extract modal parameters of structures like this bridge is by using OMA techniques. The first three calculated mode shapes and natural frequencies of this bridge are shown in Figure 1 [Ref. 1].

Over the years, OMA has evolved as an autonomous discipline. However, most OMA methods have been derived from EMA procedures, so they share a common theoretical background with input–output procedures. The main difference is in the formulation of the input, which is known in EMA, while it is random and not measured in OMA. Thus, while EMA
procedures are developed in a deterministic framework, OMA methods can be seen as their stochastic counterpart [Ref. 4].

In structural engineering, OMA is very attractive because tests are cheap and fast, and they do not interfere with the normal use of the structure. Moreover, the identified modal parameters are representative of the actual behavior of the structure in its operational conditions, since they refer to levels of vibration that are present in the structure rather than to artificially generated vibrations [Ref. 2].

The strong research activity in the field, focused on both the theoretical basis and the applications of OMA, has motivated the creation of an international conference in 2005 that is entirely focused on OMA; the International Operational Modal Analysis Conference (IOMAC) [Ref. 2].

In recent years, OMA has been widely used in civil engineering applications, eg, by extracting modal properties of towers, bridges, dams, etc. It can be an attractive method in mechanical engineering and rotating/reciprocating applications, wherever EMA is impractical or expensive. In cases where the components under study are massive or under the influence of running equipment that cannot be shut down to perform EMA, OMA can be an alternative solution.

Operational modal analysis (OMA) versus experimental modal analysis (EMA)

Experimental modal analysis (EMA)

EMA methods are generally used to determine the dynamic characteristics of small and medium-size structures. In these tests, controlled forces are applied to a structure to induce vibrations. By measuring the structure's response to these known forces, one can determine the structure's dynamic properties. The measured excitation and response time histories are used to compute frequency response functions (FRFs) between a measured point and the point of input [Ref. 2].

Forced vibrations encompass any motion in the structure induced artificially above the ambient level. The three most popular methods for testing structures are shaker, impact, and pull-back or quick-release tests. A brief description of these methods follows [Ref. 2] and [Ref. 3]:

1. Shaker tests: Shakers are used to apply forces to structures in a controlled manner to excite them dynamically. A shaker must produce sufficiently large forces to effectively excite a large structure in a frequency range of interest. For very large structures, such as long-span bridges or tall buildings, the frequencies of interest are commonly less than 1 Hz. While it is possible to produce considerable forces with relatively small shakers at high frequencies, such as those used to test mechanical systems, it is difficult to produce forces large enough to excite a large structure at low frequencies. Although it is possible to construct massive, low-frequency shakers, they are expensive to build, transport and mount. In such cases, alternative methods to excite the structure are desirable.

2. Impact tests: Impact testing is another method of forced vibration testing. In mechanical engineering applications, impact testing is commonly used to identify the dynamic characteristics of machine components and small assemblies. The test component is generally instrumented with accelerometers and struck with a hammer instrumented with a force transducer. While impact testing is commonly used to evaluate small structures, problems may occur when this method is used to test larger structures. To excite lower modes of a large structure sufficiently, the mass of the impact hammer needs to be quite large. Not only is it difficult to build and use large impact hammers with force transducers, but the impact produced by a large hammer could also cause considerable local damage to the test structure. When impact testing is performed on machinery (reciprocating, rotating equipment), the machine needs to be shut down so that the forces generated at different harmonics of the machine speed does not interfere with impact testing data.

3. Pull-back tests: Pull-back or quick-release testing has been used on some occasions to test large structures. This method generally involves inducing a prescribed temporary deformation to a structure and quickly releasing it, causing
the structure to vibrate freely. Hydraulic rams, cables, bulldozers, tugboats or chain blocks have been used to apply loads that produce a static displacement of the structure. The goal of this technique is to quickly release the load and record the free vibrations of the structure as it tends to return to its position of static equilibrium. The results from quick-release tests can be used to determine natural frequencies, mode shapes and damping values for the structure’s principal modes.

In EMA, after measuring the input and output signal, the FRF between a measured output and the point of input is computed. FRF can be used to determine the modal properties using the well-established method of EMA (see [Ref. 3] for more details on EMA methods).

In physical terms, the FRF represents the amplitude and phase of the steady-state response of a viscously damped single degree of freedom (SDOF) system subjected to a harmonic force of unit amplitude and frequency \( \omega \). We can plot the FRF as a function of the angular frequency \( \omega \). Since the function is complex, it is normally plotted in two separate plots; one plot showing the absolute value of the FRF as a function of frequency (also called the amplification function) and another plot showing the phase as a function of frequency.

The following equation shows the Fourier transform of both sides of the vibration equation for an SDOF system:

\[
(m(i\omega)^2 + c i \omega + k) Y(\omega) = X(\omega)
\]

(Eq. 1)

The FRF is:

\[
H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{(m(i\omega)^2 + c i \omega + k)}
\]

(Eq. 2)

FRF’s counterpart in the time domain is IRF (impulse response function). IRF of an SDOF of a system, generally denoted as “\( h(t) \)”, is the response when the system, is disturbed by an impulsive excitation of very short duration at time \( t = 0 \).

\[
m \ddot{h}(t) + c \dot{h}(t) + k h(t) = \delta(t)
\]

(Eq. 3)

Solving (Eq.3) for \( h(t) \) will result in:

\[
h(t) = \frac{1}{m \omega_d} e^{-\zeta \omega_d t} \sin \omega_d t
\]

(Eq. 4)

IRF and FRF are Fourier Transform pair.

**EMA with random vibration**

Random vibration deals with cases where both excitation and response are described by random processes. Neither excitation nor response signals can be subjected to a valid Fourier transform calculation, and another approach must be found. It will be necessary to introduce and to define two sets of parameters which are used to describe random signals [Ref. 3]: one based in the time domain – the Correlation Functions – and the other in the frequency domain – the spectral densities.

The correlation function is obtained by using time averaging, as shown below:
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**Equation 5**

\[ R_x(\tau) = \frac{1}{T} \int_{0}^{T} x(t)x(t+\tau)dt \]

This correlation function, unlike the original quantity \( x(t) \), does satisfy the requirement for Fourier transformation, and thus we can obtain its Fourier transform by the usual equation. The resulting parameter is called power spectral density (PSD). PSD shows the energy distribution of a signal as a function of frequency.

**Equation 6**

\[ G_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau)e^{-i\omega \tau}d\tau \]

The following equation related the input spectral density with the output spectral density:

\[ G_y(\omega) = X^*(\omega)X(\omega)H^*(i\omega)H(i\omega) = G_x(\omega)|H(i\omega)|^2 \]

(Fig. 7)

FRF can be derived from random vibration signals by estimating the relevant spectral densities.

**Operational modal analysis (OMA)**

An OMA is similar to conducting an EMA with random excitation, except that the input excitation is not measured in OMA. Moreover, unlike an EMA where the input is controlled, an OMA is based on the dynamic response of the structure under test to noncontrollable and immeasurable loadings such as environmental and operational loads. Therefore, some assumptions about the input are needed.

In OMA, ignoring the need to measure the input is justified by the assumption that the input does not contain any specific information, i.e., the input is approximately white noise [Ref. 4]. As with EMA, the measured time signals can be processed in the time domain or in the frequency domain. Since the forcing function is unknown, frequency response functions between force and response signals cannot be calculated. Instead, the analysis relies on correlation functions and spectral density functions estimated from the operational responses.

If the structure is excited by white noise, i.e., the input PSD is constant, all modes are equally excited, and the output spectrum contains full information about the structure. In this case, PSD of input is constant, and by computing the output PSD only we can derive the modal properties [Ref. 10].

However, this is rarely the case, since the input excitation has a spectral distribution of its own. Therefore, OMA-calculated modes are weighted by the spectral distribution of the white noise input and both the properties of the input excitation filter and the modal parameters of the structure are observed in the response. In this way, OMA methods can distinguish between real modes of the structure and any input excitation responses. Additionally, noise and eventual spurious harmonics due to rotating equipment are observed and removed from the OMA results. Thus, in the general case, the structure is assumed to be excited by unknown forces that are the output of the so-called excitation system loaded by white noise (Fig. 2). Under this assumption, the measured response can be interpreted as the output of the combined system, made by the excitation system and the structure under test in series, to a stationary, zero mean, Gaussian white noise [Ref. 10].

It does not matter much if the actual loads do not have exact white noise characteristics since the critical factor is that all the modes of interest are adequately excited so that their contributions can be captured by the measurements.
The application of a filter to the assumed white noise input can be described as “coloring” of the input loads. It can be proved that the concept of including an additional filter describing the coloring of the loads does not change the physical modes of the system [Ref. 2]. The coloring filter concept shows that what we are estimating in OMA is the modal model for “the whole system,” including both the structural system and the loading filter.

When interpreting the modal results, this has to be kept in mind, because, some modes might be present due to the loading conditions, and some might come from the structural system. We should also note that, in practice, we often estimate a much larger number of modes with OMA than the expected physical number of modes of the considered system. OMA, therefore, includes methods to weigh or distinguish between input-dominated modes and true natural modes of the system.

The discrimination between structural modes and properties of the excitation system is possible since the structural system has a narrowband response and time-invariant properties, while the excitation system has a broadband response with either time-varying or time-invariant properties.

Figure 2: Illustration of the concept of OMA. The nonwhite noise loads are modeled as the output from a filter loaded by a white noise load [Ref. 2].

The table below summarizes the parameters calculated in OMA and EMA:

Table 1: Comparison between EMA and OMA parameters

<table>
<thead>
<tr>
<th></th>
<th>Input</th>
<th>Time domain</th>
<th>Frequency domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMA (deterministic framework)</td>
<td>Known and measured</td>
<td>IRF</td>
<td>FRF</td>
</tr>
<tr>
<td>OMA (stochastic framework)</td>
<td>Unknown and unmeasured</td>
<td>Correlation function (CF)</td>
<td>Power spectrum density (PSD)</td>
</tr>
</tbody>
</table>

**Modal parameter estimation techniques in OMA**

In contrast to EMA, OMA testing does not require any controlled excitation. Instead, the response of the structure to “ambient” excitation sources such as wind, traffic and operating loads is recorded. As with EMA, the measured time signals can be processed in the time domain or in the frequency domain.
OMA methods based on the analysis of response time histories or correlation functions are referred to as time-domain methods. Methods based on spectral density functions are, alternatively, referred to as frequency-domain methods. This distinction may look artificial since they simply consider different representations of the same signal (in fact, it is always possible to transform the signal from one domain to the other) [Ref. 10].

Most of the classical input-output (EMA) methods can be readily modified for the operational case. For instance, frequency response function (FRF) driven methods can be converted to PSD-driven methods and impulse response function (IRF) driven methods can be used together with correlation functions instead [Ref. 10].

**Basic frequency domain (peak-picking) method**

A fast method to estimate modal parameters from OMA tests is the rather simple “peak-picking” frequency-domain technique, also known as the basic frequency domain (BFD) method. This technique has been used extensively for a variety of applications. The basic idea of the peak-picking technique is that when a structure is subjected to ambient excitations, it will have strong responses near its natural frequencies. These frequencies can be identified from the peaks in the power spectral densities (PSD) computed for the time histories recorded at the measurement points [Ref. 4]. The BFD technique can be classified as an SDOF method for OMA. In fact, it assumes that, around a resonance, only one mode is dominant. This concept is illustrated in the case study presented in this section. The significant peaks of the PSDs for the OMA measurements conducted on a 3-throw integral reciprocating compressor can be associated with the natural frequency of axial vibrations of the cylinders and suction bottle.

The inspection of the coherence functions between couples of channels also supports the identification of the actual modes of the structure. In fact, in correspondence of a resonant frequency, the coherence function is close to 1 because of the high signal-to-noise ratio at that frequency. This characteristic is helpful in the discrimination between real eigenfrequencies and peaks due to input excitation.

In principle, the BFD should be applied to evaluate natural frequencies and mode shapes only. The half-power bandwidth method is sometimes applied to get damping estimates from the spectra. However, multiple studies have shown that they are not accurate [Ref. 4].

The method identifies operational deflection shapes (ODSs) instead of the actual mode shapes, and they are generally a combination of all mode shapes; they are good approximations of the actual mode shapes if only one mode is dominant at the considered frequencies. In the case of closely spaced modes, the respective contributions are significant and the ODS is the superposition of multiple modes. Despite these drawbacks, the method is very simple and undemanding from a computational point of view. Thus, it is a basic but useful analysis tool, in particular during field tests, to get a quick insight about the effectiveness of measurements and results of dynamic identification. The BFD technique is effective when damping is low, and modes are well separated. If these conditions are violated, it may lead to erroneous results.
Case study 1: cylinder and suction bottle axial mechanical natural frequency (MNF) on a 3-throw integral compressor

This case study is about high vibrations experienced on a 3000 HP integral reciprocating compressor unit. The 3-throw, 1-stage compressor was running at 270 to 330 RPM. Suction and discharge pressure was 600 and 744 psig, respectively. High vibration on cylinders and suction bottle at speeds over 300 RPM, along with nozzle failure on the discharge bottle were experienced. Figure 3 shows a view of the compressor cylinders and suction bottles.

Site testing was done to assess vibration on the package. A vibration survey of the compressor package determined excessive axial (parallel to the compressor shaft) vibration on the compressor cylinders and suction bottle mainly at 3x (top of operating speed) and 4x (bottom of operating speed). Vibrations were measured to be 2.4 ips pk at 17.5 Hz on the cylinders. Normally, vibration levels for a compressor cylinder of this kind would be about 0.7 ips pk, about 30% of the measured vibration. ODS measurements show that all cylinders and suction bottle move in-phase in axial direction. It was suspected that a mechanical natural frequency of cylinders and bottles in axial direction is excited by pulsation-induced shaking forces mainly at 3x (top of operating speed) and 4x (bottom of operating speed).

EMA (impact testing) was conducted on the cylinders to find the axial MNF. During the impact testing, this unit was down, and an identical unit was running nearby at 300 RPM. FRF of the impact testing is shown in Figure 4. Each cylinder weights approximately 15000 lbs, and it was difficult to excite the cylinder using the impact hammer. Also, the MNF was masked by background noise and harmonic forces transmitted from the other unit. It was suspected that the peak observed at around 17.5 Hz was the MNF of cylinders and suction bottle in axial direction.

Another method that was used to detect the MNF was observing the live FFT signal, as shown in Figure 5. FFT plots at different times are superimposed. The system is very responsive at a resonant frequency. Over time, the background noise will excite the MNF, and distorted lines will form around the MNF. The nearby unit was running at 300 RPM during the bump testing, therefore harmonics of the fundamental frequency (5 Hz) showed up in the FFT plots as well.

PSD plot of random vibration collected on a cylinder in axial direction is shown in Figure 6. Maximum values of PSD occur at around 17.5 Hz. The high PSD values around this frequency is a strong indication of presence of MNF at 17.5 Hz. The PSD plot clearly shows the MNF and can be used instead of FRF or FFT plots. The natural frequencies can be identified from the peaks in the power spectral densities (PSD) computed for the time histories recorded at the measurement points, with random vibration data. This method is BFD and was discussed in the previous section.

Results from finite element modeling confirmed an axial mode of suction bottle and cylinder at 17.3 Hz (Figure 7).
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Figure 4: Case study 1, cylinder axial impact test FRF. The results show excitation at different harmonics (1x, 2x, etc.) from a unit running nearby at 300 RPM as well as cylinder MNF

Figure 5: Case study 1 – live FFT signal of axial vibration on the cylinder, random background noise (nearby unit running at 300 RPM)
Figure 6: Case study 1 – PSD plot of random vibration data collected on cylinder and suction bottle during unit down time

Figure 7: Case study 1, axial mode of cylinders and suction bottle at 17.3 Hz, computed in FEA
The frequency domain decomposition (FDD) method

FDD is an OMA method in the frequency domain. OMA methods in the frequency domain are based on the simple relation between input and output power spectral density (PSD) of a random process. The simplest technique in the frequency domain is the BFD technique discussed in the previous section.

If the modes are well separated, this technique will lead to acceptable estimates. The main advantages of this technique are the absence of computational modes as well as the simplicity and speed of the method. However, BFD has its own problems. This includes low accuracy, especially for complex structure due to the dependency of the result to the resolution of PSD spectrum, low accuracy in calculation of damping ratio and inapplicability for systems with close modes [Ref. 11].

With the idea of proposing a method in frequency domain, having the above-mentioned advantages and relieving those discrepancies, Brincker et al. [Ref. 2] presented the frequency domain decomposition (FDD) technique. In this technique, singular value decomposition (SVD) of the output PSD matrix in different frequencies is used as the mode indicator function (MIF) [Ref. 11].

In addition to identifying the modes, especially the close modes, the noise and signal space get separated in this technique. By using the FDD method, the natural frequencies and mode shapes can be obtained.

For estimating damping coefficients, a method named enhanced frequency domain decomposition (EFDD) was introduced by Brincker et al. [Ref. 2] . In this method, the singular values in the proximity of natural frequencies are transferred to the time domain by inverse FFT, and damping coefficients are obtained using logarithmic decrement techniques. The theoretical background for these different techniques is not discussed in this paper. See [Ref. 2] and [Ref. 4] for theoretical details about these techniques.

The time domain decomposition method

In time domain identification, all the techniques used in a classical modal analysis based on impulse response functions can be used in OMA on correlation functions. In the time domain, we are dealing with free decays – in OMA normally estimated as correlation functions or similar functions such as random decrement functions.

A number of modal identification techniques, initially developed in the deterministic framework of traditional input-output modal analysis, have been extended to output-only modal analysis recognizing that the correlation functions of random responses can be expressed as a sum of decaying sinusoids holding the information about the modal parameters. As a consequence, correlation functions of the random responses of the structure under natural excitation have replaced the experimental estimates of IRFs in the application of such modal analysis techniques in the output-only case. For this reason, this class of OMA methods is traditionally referred to as natural excitation technique (NExT) [Ref. 2].

Each decaying sinusoid has a damped natural frequency, damping ratio and mode shape coefficient which is related to one of the structural modes. Therefore, in the time domain EMA techniques, instead of impulse response function (IRF) in MIMO (multi-input multi-output) systems, correlation functions (Eq. 5) can be used to obtain the modal parameters of the system. This, indeed, provides a good ground to extend and use the EMA techniques in operational modal analysis.

One of the modal identification methods in the time domain that has been developed and applied in OMA is a stochastic subspace-based method. In the 1990s, in the field of systems and control engineering, a new subspace-base method for identification of systems state space was proposed which directly used the measured data [Ref. 11].
Case study 2: foundation resonance

This case study involves the troubleshooting of high vibration on a 2-throw, 4-stage compressor package on a driven steel pile foundation. The package owner had experienced high vibration on the compressor cylinders, bottles and piping when operating at speeds around 1,080 RPM. Figure 8 shows a plan view of the compressor package.

Site testing was done to assess vibration on the package. A vibration survey on the compressor package determined excessive vertical vibration on the compressor cylinders at 2X compressor speed. Vibrations were measured to be 4.7 ips pk on both cylinders. Normally, vibration levels for a compressor cylinder such as this would be about 0.7 ips pk, about 1/7 of the measured vibration [Ref. 5].

Vertical vibrations on the piles were found to be 0.6 ips pk at 36 Hz for the worst case. A typical skid vibration guideline is 0.1 ips pk. The excessive vibration measured on the skid beams in the vertical direction lead to the initial conclusion that there may have been a poor connection of the skid to the pile. Measurements on the top of the pile and bottom of the skid were taken. The time trace of the vibration at the top of the pile and bottom of the skid almost exactly follow each other. This indicates the skid is well connected to the pile. A vertical vibration mode of the foundation combined with the package was suspected to be excited by cylinder vertical shaking forces.

Design work was undertaken to determine if the model could calculate the observed behavior and then be used to determine possible solutions to reduce vibration. Pulsation-generated shaking forces are calculated in the piping system and pulsation bottles. One shaking force that is often overlooked is the shaking force in the vertical direction that is generated between the compressor cylinder and pulsation bottle, as shown in Figure 9. A net force in the vertical direction results from the difference in dynamic pressure at the suction bottle outlet nozzle at the shell and the area projection on the gas passage. A shaking force is generated between the suction bottle and cylinder as well as between the discharge bottle and cylinder. These two forces add to create a total shaking force on the cylinder. The calculated shaking force acting on the compressor cylinder for the field test condition is as determined by the pulsation model shown in Figure 10. The force is relatively high, up to 4,000 lb pk-pk at 2X compressor speed. A typical guideline used by
Wood for these types of compressor is 1,000 lb pk-pk. This shaking force is excessive and the likely cause of the high cylinder and foundation vibration.

![Figure 9: Shaking force between the suction bottle and cylinder](image)

![Figure 10: Cylinder 1 shaking force](image)
A model of the existing compressor package and foundation was developed. The model included simulation of the pile and soil dynamic properties, skid structure, detailed model of the compressor, bottles and piping, motor and cooler. All the dynamic forces generated by the compressor, including unbalance, crosshead guide forces, internal cylinder gas forces, and pulsation shaking forces were applied to the model. Figure 11 shows a vector plot of the calculated vibration at 36 Hz. The model results agree well with the field measurements in both the frequency of the highest vibration as well as amplitude.
Figure 11 – Calculated vibration at 36 Hz
OMA to identify modal properties

OMA techniques were used to measure and identify the modal properties of the foundation. Sensors were installed on the piles underneath the skid to measure vertical vibrations of the foundation (Figure 12). A commercial software package (ARTeMIS) was used to perform OMA on the collected data. Figure 13 shows the mode shape of the skid calculated using FEA and OMA. Modes of the foundation were estimated using a frequency domain (EFDD). Results are shown in Figure 14. In Figure 14, singular values of the spectrum-density matrix are plotted for all test points. Modal coherence and modal domain are used as indicators to distinguish between real structural mode peaks and disturbance (noise) peaks in spectrum density plots. The theory of these indicator functions is described in [Ref. 8] and is summarized below:

Modal coherence indicator

Modal coherence indicator function denotes the correlation between the first singular vector at a selected frequency \( f_0 \) and a neighboring frequency \( f \). If the modal coherence is close to unity, then the first singular value at the neighboring point corresponds to the same modal coordinate as at \( f_0 \), and therefore the same mode is dominating. If it is close to 0, then the neighboring singular vectors are not correlated and therefore are not dominated by any mode but by noise instead [Ref. 12]. In Figure 14, a large amount of blue from the top means low coherence and hence a lot of noise, whereas no blue indicates good coherence between neighboring frequency lines.

Modal domain indicator

The formula for the modal domain indicator function is similar to the modal coherence function, except that instead of being a function of the initial point given by \( f_0 \), it is a function of the frequency \( f \) of the considered neighboring points. Around a structural mode peak at \( f_0 \), the correlation should be high for a larger frequency range where that mode is dominating. This range is called the modal domain and no second mode peak is considered for automatic mode selection inside that domain after the automatic peak selection has chosen the first one [Ref. 12]. In Figure 14, the modal domain is shown by green areas.

As can be seen in Figure 14, two modes of the structure have been detected by OMA, first mode at 27 Hz (Rocking mode) and second mode at 36 Hz (Vertical mode). The vertical mode is of interest for this case study.
Figure 12: Physical layout and test point locations on piles – case study 2
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Figure 13: Foundation vertical mode shaped – case study 2

Figure 14: Modal estimation- using frequency domain algorithm – case study 2
Case study 3: pipeline resonance

This case study is about high vibration on a 48" natural gas pipeline, which was the discharge header of a centrifugal compressor driven by a gas turbine. Piping was supported on HSS beams (6"x6"x3/8"), 7 feet height. High vibration on the piping (test point TP-D4) and support structure (test points SS-1 to SS-3) were observed. Figure 15 shows the vibration test points on piping and support structure. Vibrations were measured to be 2.3 ips pk on the piping and 1.9 ips pk on the support structure. Normally, vibration levels for piping at the measured frequency would be about 0.35 ips pk based on VDI 3842 guideline (correction), about 15% of the measured vibration.

Dynamic pressure measurements show a broadband low-frequency excitation, which is indicative of flow-induced turbulence (FIT). Turbulent energy is created in the piping system when fluid flows past flow discontinuities in the system, due to high fluid kinetic energy. The majority of the turbulent energy concentrates at low frequency (below 50 Hz in this case). The level of excitation was higher at lower frequencies.

Figure 15: Physical layout and test point locations on piping and support structure – case study 3
The broadband dynamic pressure generates shaking force at elbows and reducers. The shaking forces excite all low-frequency modes of the piping and support structure. The piping system, on both suction and discharge line, has several low-vibration modes due to flexible supports. The static stiffness of pipe supports in axial (flow direction) and horizontal direction was calculated using FE analysis (Figure 16). A typical effective pipe support static stiffness is 1e5 lbf/in.

![Figure 16: Pipe support static stiffness in axial and horizontal direction – case study 3](image)

FE work was undertaken to determine if the model could calculate the observed behavior and then be used to determine solutions to reduce vibration. A model of the existing piping and support structure and foundation was developed. The model included a simulation of the pile and soil dynamic properties. The modal results show an axial mode of piping and support structure excited by flow-induced shaking forces (Figure 17).

OMA was used to calculate modal properties of the axial mode of piping. Data was captured on a pipe support structure at three test points (SS-1, SS-2 and SS-3 as shown in Figure 15) during random (ambient) excitation. Figure 18 shows the OMA results performed on the discharge piping. A mode at ~3 Hz was detected in which all test points are moving in phase. The estimated damping ratio is 1.6% for this mode. FE results (mode shape and frequency) are in close agreement with the measured properties using OMA. A frequency-domain (EFDD) algorithm was used in this case to estimate modal properties.
Figure 17: Discharge piping axial mode shape – case study 3

Figure 18: Estimated modal properties of discharge pipe supports, using OMA – case study 3

Lowest mode at ~3 Hz, damping ratio: 1.6%
Performing OMA in the field

Despite the differences in excitation, OMA testing consists of the same three basic steps encountered in EMA:

- Planning and execution of tests: this step concerns the definition of the experimental setup (measurement chain, sensor layout, attachment of sensors, cable paths, etc) and the data acquisition parameters (duration of records, sampling frequency).
- Data processing and identification of the modal parameters: this step concerns the validation and pre-treatment (filtering, decimation, etc.) of the acquired data, some signal processing operations (for instance, for the computation of correlation functions, PSD functions, etc), and the estimation of the modal parameters.
- Validation of the modal parameter estimates

All current techniques for operational identification can be formulated for multiple inputs, and to clearly identify closely spaced modes, the loading, in fact, must be multiple-input. When applying OMA, it can be challenging to ensure multiple-input loading since the input forces on the structure are not measured. One class of loading that is clearly multiple-input includes ambient excitation such as loading over a large part of the structure, such as traffic, wind, machinery operating loads, etc.

Number and locations of sensors

The initial task in dynamic measurement planning is the selection of the sensor locations and directions, which determines the total number of measurements. The choice can be based on experience, on computer simulations using finite element models of the structure or on predictions of the dynamic response of the structure based on simple beam, plate or shell theories. For all types of measurements, each transducer should be as small and lightweight as possible in order to minimize the influence of the added mass from the sensors. The sensors should also be sensitive enough to pick up the expected operating signals.

The easiest way to understand how many sensors are needed for an OMA test is to consider the “rank of the problem” in connection to the frequency-domain decomposition (FDD) theory, the details of which are presented in [Ref. 2 and 4].

It is important to understand that the measurement system should have the ability to clearly identify all important sources of noise and physical responses in such a way that the measurement system itself does not limit the rank of the problem. If two measurement points are close to each other, they represent the same information, and thus, the added channel does not in practice contribute to the matrix rank. Therefore, it is essential that the number of measurement channels is larger than the rank of the problem defined earlier, and secondly, that the measurement points are spread over the structure so that each individual measurement point does not just mainly repeat information in other channels. It is also important to avoid placing sensors in node points of interest, as this would prevent it from obtaining information about this mode.

One should never assume the rank of the problem to be smaller than four (See [Ref. 2] for theoretical background). To perform a successful OMA testing and analysis we recommend placing at least four sensors at ‘correct’ locations indicated above However, due to the inherent uncertainty on the correct sensor locations away from any node points, it is good practice to use at least five or six sensors [Ref. 2].

Simulations

One of the best ways to determine correct sensor locations is to perform simulations. Since the exact properties of the structure under consideration are unknown, we need to simulate a set of possible responses using different models and different placement of the sensors and see which locations provide the best estimates for all models considered.

Frequency ranges

After selecting measurement numbers, locations and directions, the next task is to establish the frequency range for each measurement, that is, the maximum and minimum frequencies to be recorded and analyzed. The selection of the maximum frequency is often the most challenging part of this task; it generally has the greatest impact on the total data
or system bandwidth and on the various instruments of the measurement system. The maximum frequency is usually selected to equal the highest of the following:

- The maximum significant frequency of the operating response of the structure
- A standard frequency for the data acquisition system, for instance, systems for testing of large structures are often having a standard Nyquist frequency of 50 or 100 Hz

**Measured duration**

It is always desirable to measure the response of a structure during the complete duration of a specific excitation. For random signals, the choice of recording duration depends on the choice of permissible errors. The following equation can be used as a good estimate for the length of time series required to perform OMA [Ref. 2]:

\[ T_{tot} > \frac{10}{\zeta f_{min}} \]  
(Eq. 8)

- \( T_{tot} \): Total time series length
- \( f_{min} \): lowest expected natural frequency of the system
- \( \zeta \): damping ratio

For piping and structures (skid, platform, etc), a typical damping ratio is 1-2%. Therefore, if for example, we expect to have natural frequencies as low as 5 Hz on a mechanical system, then 200 seconds of data will be sufficient to perform OMA.

**Detecting harmonics**

As previously noted, the theoretical framework of OMA depends on the assumption that excitation forces consist of white noise. This can be considered true for many ambient processes which have stochastic nature. However, in mechanical structures typically include rotating and reciprocating equipment such as compressors, engines, generators or turbines. These devices excite the structure with harmonic loads and influence the response frequency spectrum significantly. Thus, algorithms are needed to detect and eliminate those influences. There are techniques and commercial software packages that would eliminate harmonic influences during the mode estimation. One common method is based on the probability density function (PDF) calculation [Ref. 6]. Structural modes as a response to stochastic have a PDF of Gaussian distributed bells. Harmonic excitation, on the other hand, produces a very different PDF (Figure 19). See [Ref. 2] and [Ref. 9] for theoretical background on this topic.
Mathematical methods are typically used to produce a robust algorithm for automatic detection. Still, the analyst should verify the data and use their experience to distinguish between structural modes and harmonic excitations:

- A narrow peak in more than one singular value (SVD) indicates a harmonic excitation in contrast to a peak of a structural mode in only one singular value. If a harmonic is present, it excites all existing modes to the extent of their participation factor. Thus, all singular values show a peak at the frequency of the harmonic excitation.

- When being animated, an estimated structural mode shape should be shown as a “normal” mode. In a normal mode, all parts of the structure have a coherent phase angle, so the mode shape vector can be expressed without imaginary components. The mode is animated with stationary nodal lines. On the other hand, if a peak in the frequency plot has been selected, which is due to harmonic excitation instead of a structural mode, it is likely that more than one structural mode participates at that frequency. This results in imaginary components or different phase angles of channels. Such a “complex” mode has non-stationary nodal lines in the animation and is not a distinguished structural mode with modal parameters.

- Very low calculated damping ratios may be due to a harmonic excitation constantly feeding the structure with power.

**Modal validation**

The same techniques that are typically used in EMA can be used in OMA. In this section, mode complexity and modal assurance criteria are discussed, and results are presented for case study 2 and 3. Some other techniques such as modal coherence and modal domain in the frequency domain are used as well in some algorithms in order to distinguish the real modes from noise or harmonic excitation (e.g., machine running). See [Ref. 3], [Ref. 6] and [Ref. 7] for more details and theoretical background.

**Mode complexity [Ref. 7]**

For most structures of, e.g., concrete and steel, a proportional damping model is a good approximation. In such a case, the mode shapes are real-values meaning that minimum and maximum values are occurring at the same time during a mode shape animation.

If mode shapes are complex, it can be due to one or more of the above reasons:
• Non-proportional damping
• Bad measurements or poor modal parameter estimation
• Inconsistent data due to e.g., time-variant conditions

In a case like this, the mode shape animation in the geometry window will show a “traveling wave” due to minimum and maximum values not occurring at the same time.

Another way to view this is to look at all the mode shape components in the complex plane. Such a diagram is called a complexity plot and is shown below for a certain mode.

Each mode shape component is represented by a vector starting in 0,0 and the pointing out to the mode shape components real and imaginary value. If the component is real-valued, it should point in a horizontal direction, so complexity is illustrated by the vertical component.

**Figure 20** shows the complexity plots for the estimated modes in case study 2 and 3. The low complexity of the test points for these modes, i.e., most vectors close to horizontal (real) axis confirms the good quality of data and OMA analysis.

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**Figure 20: Complexity plot for the estimated mode in case study 2 and 3**

**Modal assurance criteria (MAC) [Ref. 13]**

The modal assurance criterion analysis (MAC) analysis is used to determine the similarity of two mode shapes. The MAC value between two modes is essentially the normalized dot product of the complex modal vector at each common node (i.e., points).

If a linear relationship exists (i.e., the vectors move the same way) between the two complex vectors, the MAC value will be near to one. If they are linearly independent, the MAC value will be small (near zero).

A complex vector simply includes both amplitude and phase. In Equation 1, it is also clear that the MAC is not sensitive to scaling, so if all mode shape components are multiplied with the same factor, the MAC will not be affected.
If an experimental modal analysis had 20 different measured nodes, the mode shape components at all 20 nodes are taken into account to calculate the MAC value, but more importance will be attributed to the higher-amplitude node locations.

A modal assurance criterion (or MAC) analysis can be used in several different ways:

- **FEA-test comparison** – a MAC can be used to compare modes from an experimental modal analysis test to a Finite Element Analysis (FEA) and an object. It will indicate if the same mode shapes are found in both the test and FEA analysis.
- **FEA-FEA comparison** – several assumptions can be made in the creation of an FEA analysis: Young’s Modulus, boundary conditions, and mass density values, to name a few. A MAC analysis can determine the degree to which these assumptions affect the resulting mode shapes.
- **Test-test comparison** – a MAC analysis can flag potential issues with the modal analysis results. Usually, MAC will identify modes and areas that could benefit from acquiring more data points on the structure.

*Figure 21* shows the MAC values the estimated modes in case study 2 and 3. Diagonality of MAC values, i.e., close to 1 for diagonal elements and close to zero for other elements confirm the good quality of data and OMA analysis.

![Figure 21](image-url)

**Conclusions and recommendations**

The operational modal analysis technique allows a scientist, technician or engineer to perform a modal investigation easily, quickly and accurately. It can be accomplished by only measuring the response of the structure subjected to unknown and unmeasured input forces. Impact or shaker testing are still valid and accurate methods to perform modal testing on small to medium-size structures. OMA can be used in cases where EMA fails to provide reliable data or is expensive or difficult to perform.
Glossary

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>BFD</td>
<td>Basic frequency domain</td>
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<tr>
<td>EFDD</td>
<td>Enhanced frequency domain decomposition</td>
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<td>EMA</td>
<td>Experimental modal analysis</td>
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<tr>
<td>FDD</td>
<td>Frequency domain decomposition</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analysis</td>
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<td>FFT</td>
<td>Fast Fourier transform</td>
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<tr>
<td>FRF</td>
<td>Frequency response function</td>
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<tr>
<td>IOMAC</td>
<td>International operational modal analysis conference</td>
</tr>
<tr>
<td>IRF</td>
<td>Impulse response function</td>
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<tr>
<td>MAC</td>
<td>Modal assurance criteria</td>
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<tr>
<td>MIF</td>
<td>Mode indicator function</td>
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<tr>
<td>MIMO</td>
<td>Multi-input multi-output</td>
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<td>MNF</td>
<td>Mechanical natural frequency</td>
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<td>ODS</td>
<td>Operating deflection shape</td>
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<td>OMA</td>
<td>Operational modal analysis</td>
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<tr>
<td>PSD</td>
<td>Power spectrum density</td>
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<tr>
<td>SDOF</td>
<td>Single degree of freedom</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular value decomposition</td>
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</tbody>
</table>

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