SIMPLIFIED PREDICTION
OF
BALANCE SENSITIVITY

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ABSTRACT

It is possible to estimate the first and second critical speeds of a rotor by making use of measurements during a coast-down.

Combining the knowledge of the critical speeds with the geometry and weight of the rotor, it is possible to estimate the sensitivity to static and dynamic unbalance.

The rotor should be operating in the “rigid rotor” region as opposed to the “flexible rotor” region.

BIOGRAPHICAL SKETCH

Brian C. Howes, M.Sc., P. Eng., is the Chief Engineer at Beta Machinery Analysis Ltd. in Calgary, Alberta, and has been involved in machinery troubleshooting for both rotating and reciprocating equipment since 1972. He is one of the foremost experts in vibration analysis.

Scott Masterson is also a Project Engineer with Beta Machinery Analysis Ltd.

INTRODUCTION

This paper describes a technique for balancing machines at speeds between their first and second shaft criticals in a single run.

When a machine has no balancing history, conventional balancing procedures will often require two or three trial weight runs to determine sensitivities and to balance the rotor. Unfortunately, costly down-time often prevents the completion of such a job.

The method of STATIC – DYNAMIC BALANCING uses the following assumptions to simplify the problem:

1. The rotor and its supports are approximately symmetrical about the centre of mass.
2. Two plane balancing is being performed.
3. Static and dynamic components of vibration are being considered as opposed to the “raw” vibration.
4. The machine approximates a rigid rotor/flexible bearing system. (Refer to Figure 1)

The experimental technique requires the collection of “as found” and “coast-down” vibration data. These data, along with some of the physical characteristics of the machine, will enable a calculation of both the static and dynamic balance sensitivities of the system. Some sample calculations will follow.

The reasoning behind this type of balancing is that a two plane balancing problem can be effectively reduced to two single plane problems that can be solved simultaneously in one run.
PROCEDURE

Static – Dynamic balancing proceeds as follows:

1. Collect “As Found” vibration data.
   - Proximity probes give the best results for rigid pedestals.
   - Shaft absolute (Shaft stick) data is more appropriate when pedestal vibration is significant.

2. Obtain displacement amplitude and phase data from the coast-down.

3. After performing runout subtraction, resolve the “as found” and coast-down data into static and dynamic components. (Refer to: Rotor Balancing by Static-couple Derivation, John M. Csokmay).

4. Construct Bode or Nyquist plots of both the static and dynamic components of the rundown.

5. Determine the first (static) critical from coast-down data, and ensure that the operating speeds is below the second (dynamic) critical.

6. Estimate the total rotating mass.

7. Calculate the theoretical static and dynamic balance sensitivities (See calculations that follow procedure.)

8. Calculate both the static and dynamic masses required at each balance plane from the sensitivities.

9. Determine the angles by which the high spots lag the heavy spots from the two coast down plots. Locate the light spot for both static and dynamic unbalances on each balance plane.

10. Vectorially sum the masses required at each balance plane and attach the two resultant masses at the required locations.

CALCULATING STATIC AND DYNAMIC BALANCE SENSITIVITIES

This method operates under the premise that both static and dynamic (couple) unbalances are independent and have separate balance sensitivities. These sensitivities are calculated as follows:

Static Sensitivity

The force from static unbalance mass, \( m_s \), rotating at a radius, \( r \), and speed, \( w \)

Equals:

The force of the entire rotating mass, \( M \) rotating about an eccentric distance, \( e_s \) at speed, \( w \).

Or:

\[
(m_s)(r)(w)^2 = (M)(e_s)(w)^2
\]

The static sensitivity can be given in the following terms:

\[
S_s = (m_s)(r)(e_s) = M
\]
Static Sensitivity: (continued)

The force from the dynamic unbalance mass, \( m_d \) rotating at a radius, \( r \) and speed, \( w \)

Equals:

The dynamic force opposing the stiffness, \( K \) at each bearing.

The total bearing stiffness is calculated as follows:

\[
W_c = \left( \frac{K}{M} \right)^{1/2}
\]

\[
K = M w_c^2
\]

But stiffness at a bearing is \( K/2 \)

The dynamic force at the bearing opposing this stiffness is:

\[
F_d = \frac{K}{2} (e_d) = 0.5 M (e_d) w_c^2
\]

The force from the dynamic unbalance mass at each balance plane is:

\[
F_d = (m_d) (r) w^2
\]

But the dynamic forces at the bearings are reduced by the ratio of \( L_1/L_2 \):

\[
F_d = F_d' \left( \frac{L_1}{L_2} \right)
\]

Let: \( c = \left( \frac{L_1}{L_2} \right) \)

\[
F_d = c (m_d) (r) w^2
\]

Equating Forces:

\[
c (m_d) (r) w^2 = 0.5 M (e_d) w_c^2
\]

The dynamic sensitivity is then:

\[
S_d = \left( \frac{m_d}{(m_d) (r) (e_d)} \right) = \frac{M w_c^2}{2 c w^2}
\]
Estimate of Balance Sensitivities:

Static Sensitivity:

\[
S_s = \frac{(m_s)(r)}{(e_s)} = M
\]

\[
S_s = \frac{11,800 \text{ kg in/in pk}}{416 \text{ oz-in/mil pk}} = 208 \text{ oz-in/mil p-p}
\]

This is the total sensitivity.

104 oz-in/mil p-p will be required at each balance plane.

(manufacturers rated static sensitivity is 200 oz-in/mil p-p total)

Dynamic Sensitivity:

\[
S_d = \frac{(m_d)(r)}{(e_d)} = \frac{M w_c^2}{2 c w^2}
\]

\[
S_d = \frac{(11,800 \text{ kg})(1350)^2}{2(.67)(3600)^2}
\]

\[
S_d = 1240 \text{ kg in/in pk} = 43.7 \text{ oz-in/mil pk} = 21.8 \text{ oz-in/mil p-p}
\]

This is the sensitivity at each bearing. 21.8 oz-in/mil p-p will be required at each balance plane.

(manufacturers rated dynamic sensitivity is 14-21 oz-in/mil p-p)

EXAMPLE 1:

SAMPLE BALANCE CALCULATIONS FOR A 30 MW GENERATOR

Total rotor mass: \( M = 11,800 \text{ kg} \)
Operating Speed: \( w = 3600 \text{ rpm} \)
1\text{th} Critical Speed: \( w_c = 1350 \text{ rpm} \)
Radius for Balance Weights: \( r = 14" \)
\( L_1/L_2 \) ratio: \( c = 0.67 \)
Figures 5 and 6 demonstrate the static and dynamic components of the generator coast-down vibration. Data have been taken with proximity probes, and slow roll corrected. The angles by which high spots lag the heavy spots can be found from the total phase changes over the coast-down.

**EXAMPLE 2: COAL DRYER EXHAUST FAN**

**Machine Layout**

- Total Rotor Mass \( M = 10,200 \) Kg
- Operating Speed \( w = 1190 \) RPM
- First Critical Speed \( w_c = 880 \) RPM
- Radius for Balance Wts. \( R = 48" \)
- \( L_1/L_2 \) Ratio \( c = .22 \)

**Estimate of Balance Sensitivities**

Using the same calculations as Example 1, the following sensitivities are obtained:

- Estimated Static Sensitivity: \( S_s = 180 \) oz-in/mil p-p
- Estimated Dynamic Sensitivity: \( S_d = 224 \) oz-in/mil p-p

**Estimate of Balance Weight Placements**

Figures 8 and 9 show static and dynamic rundown plots. The static lag angle (high spot lags heavy spot) is about 130° and the dynamic lag angle is about 75°.

The static and dynamic weights should be placed opposite the respective heavy spots.
Resolved As Found Case Vibrations:

Static Component = 0.90 mils p-p, 143°
Outboard Dynamic Component = 1.14 mils p-p, 99°
Inboard Dynamic Component = 1.14 mils p-p, 279°

B. Attach Balance Weights

These balance weights were determined from previously known overall sensitivities before the rotor mass was known.

Total Static Weight = 101 g
Dynamic Weight = 123 g per side

Vibration after placement of balance weights:
Static Component = .38 mils p-p, 179°
Outboard Dynamic Component = .30 mils p-p, 348°
Inboard Dynamic Component = .30 mils p-p, 168°

C. Weight Effect Vectors, T.

ie: \[ T = \text{Effect of Weights} = (\text{Trial weight vibration}) - (\text{As found vibration}) \]

\[
T_{\text{Static}} = .38, 179° - .90, 143° \\
= 0.63 \text{ mils p-p, 302°}
\]

\[
T_{\text{Dynamic Outboard}} = .30, 348° - 1.14, 99° \\
= 1.28 \text{ mils p-p, 292°}
\]

Calculation of Actual Sensitivities:

Static Sensitivity, \( S_s = \frac{M_r}{T_{\text{Static}}} \)

\[ = 101 \text{ g} \times 48\" / 0.63 \text{ mils p-p} \]

\[ = 7695 \text{ g-in/mil p-p} \]

\[ S_s = 271 \text{ oz-in/mil p-p} \]

Dynamic Sensitivity, \( S_d = \frac{M_r}{T_{\text{Dynamic}}} \)

\[ = 123 \text{ g} \times 48\" / 1.28 \text{ mils p-p} \]

\[ = 4613 \text{ g-in/mils p-p} \]

\[ S_d = 163 \text{ oz-in/mils p-p} \]

Actual Balance Sensitivities

A. As Found Case Vibrations:

Fan Outboard = 1.91 mils p-p, 118°
Fan Inboard = .80 mils p-p, 227°
EXAMPLE 3: ROTOR MODEL

Here is an example of a non-symmetrical rotor with a very flexible shaft. The layout is shown in Figure 10.

![Figure 10 Rotor Model](http://www.BetaMachinery.com)

M = 1612 g  w = 5000 rpm
m₁ = 629 g  w₁ = 3500 rpm
m₂ = 816 g  r = 1.2"  c = .66

Estimated and actual sensitivities are as follows:

Static Sensitivity:
- Estimated, \( S_s = 0.0284 \text{ oz-in/mil p-p} \)
- Actual, \( S_s = 0.0177 \text{ oz-in/mil p-p} \)

Dynamic Sensitivity:
- Estimated, \( S_o = 0.0102 \text{ oz-in/mil p-p} \)
- Actual, \( S_o = 0.0289 \text{ oz-in/mil p-p} \)

The discrepancies between actual and estimated values result from the fact that the shaft is neither rigid or symmetrical.

It should be noted that there are additional sources of dynamic unbalance when the balance weights at a plane are not symmetrically applied.