SENSITIVITY OF TORSIONAL ANALYSES TO UNCERTAINTY IN SYSTEM MASS ELASTIC PROPERTIES

Brad D. Murray, M.Sc., P.Eng.

Brian C. Howes, M.Sc., P.Eng.

Jonathan Chui, B.Sc., E.I.T.

Valerie Zacharias, M.A.

Beta Machinery Analysis Ltd.
300, 1615 – 10th Ave. S.W.
Calgary, AB, Canada T3C 0J7

ABSTRACT

Shaft torque and corresponding stresses in a motor driven –direct coupled –reciprocating compressor system, are significantly influenced by the system's mass-elastic properties. Uncertainty in the system's mass-elastic properties will therefore translate into uncertainty in the calculated system stresses. Three case studies provide the reader with an appreciation for the importance of defining uncertainty bands in the mass-elastic properties of a torsional system.

The case studies are followed by a theoretical discussion section. The reader is introduced to the concept of torsional resonance, uncertainty in calculated system torsional natural frequencies is defined, and the relative influence that specific system mass-elastic properties have on the overall system is discussed. A full compliment of system mass-elastic uncertainties is presented.

DATA UNCERTAINTY – KEY TO AN ACCURATE TORSIONAL ANALYSIS

Many pipelines use reciprocating compressors driven by fixed speed electric motors. Any such system has inherent torsional vibration which, if not properly addressed, can result in catastrophic failures.

Torsional vibration creates alternating stresses in the system's shafts. Accurate prediction of the overall system torque and corresponding shaft stresses requires both mean and dynamic torques. The mean torque is readily calculated based upon the system power. A detailed forced response analysis is required to calculate dynamic torque.

A forced response analysis can be very accurate, but is highly sensitive to system torsional natural frequencies (TNF's), and therefore to uncertainty in the system mass-elastic properties. The mass-elastic properties are carefully determined by the respective manufacturers; nevertheless there is a finite degree of uncertainty. It is these uncertainties in the mass elastic properties which must be quantified and converted to tolerances in the predicted system natural frequencies. The forced torsional responses, as calculated over this TNF uncertainty band, will then be representative of the "real" system.

Having established a torsional natural frequency (TNF) uncertainty band, the "design TNF" which corresponds to the highest torque can be identified. There will be confidence that all potentially critical (i.e. resonant) torsional responses will have been addressed by the modelling. Furthermore, if the TNF's have to be repositioned so as to minimize dynamic torsional amplification, the entire uncertainty band could potentially be placed *between* order of run speed, thereby avoiding resonance.

CASE STUDIES

Case 1 – System Resonance Causes Auxiliary Drive Failures: Erroneous Mass-Elastic Data

Reciprocating compressor systems often include a lube oil pump driven from the opposite drive end (ODE) of the compressor shaft. This particular system was comprised of a

- six throw reciprocating compressor operating at 885 RPM – driving an auxiliary lube oil pump, a
- flexible disc coupling, and a
- 3000 HP squirrel cage induction motor.

Torsional vibration caused the oil pump drive shaft to fail within one hour of service. The pump was subsequently replaced but again failed within one hour of service.

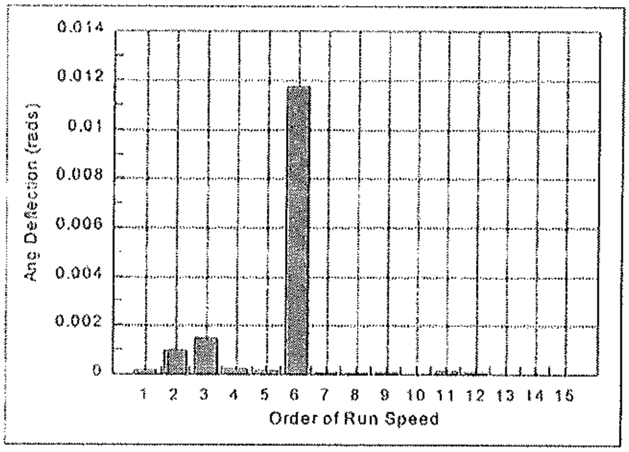

Figure 1, Torsional response at the compressor ODE was excessive at the 6th order of run speed.

A preliminary analysis indicated that the first torsional natural frequency (TNF) of 80.5 Hz (5.5x run speed) was appropriately positioned between the fifth and sixth order resonance, results in minimum dynamic torsional amplification. However as shown in Figure 1, the measured sixth order torsional response was excessive; more than 8 times greater than the third order response. This difference and the fact that the sixth order torque effort (i.e. torsional excitation) is less than the third order torque effort (Figure 2), leads to the conclusion that the first torsional natural frequency is relatively close to the sixth order of run speed (88.5 Hz).

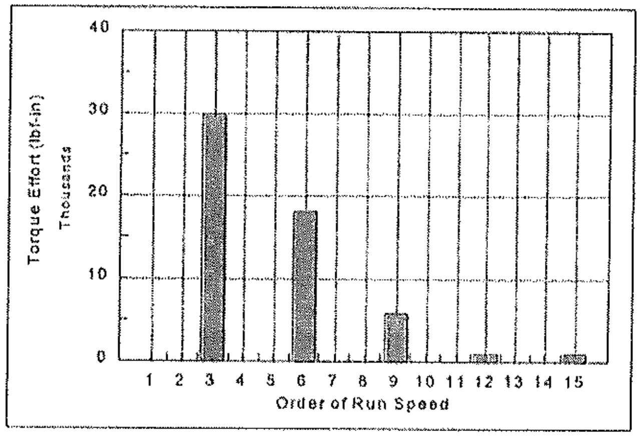

Figure 2, Third order torque effort is greater than the 6th order torque effort (i.e. torsional excitation).

Based on field run-down tests, the system's first TNF was determined to be approximately 87 Hz (5.9x run speed), 6.5 Hz greater than predicted. This discrepancy between the predicted and measured first TNF was the result of both erroneous system mass-elastic data and a large uncertainty band. That is:

i) The motor shaft had been positioned approximately 6" beyond the coupling hub (i.e. extended into the coupling center section), thereby significantly increasing the system torsional stiffness.
ii) the coupling was torsionally stiffer than originally indicated, and
iii) uncertainties in the motor mass-elastic characteristics were significant.

As well as quantifying the system mass-elastic and thereby system TNF uncertainties, the motor shaft length and coupling stiffness were corrected in the model. The theoretical model was effectively validated by the fact that the measured TNF fell within the newly calculated TNF uncertainty band. The theoretical model could then be confidently used to calculate the additional compressor inertia [1] required to minimize interference of the first TNF with the sixth order torque effort.

Although the auxiliary oil pump is relatively insignificant in terms of the capital costs, it is a critical system component; an oil pump failure could bring an entire system "down". Auxiliary driven equipment such as lube oil pumps should be considered in a system torsional analysis.

Case 2 – Undesirably High System Torsional Natural Frequency: Torsionally Stiff Motor Rotor Shaft

Rotor inertial and shaft stiffness of large electric motors, have a significant influence on the system mass-elastic characteristics. This particular system was comprised of a

- four throw reciprocating compressor operating at 895 RPM,
- flexible disc coupling, and a
- 2200 HP squirrel cage induction motor.

Preliminary calculations indicated that the first system TNF was close to the sixth order of run speed. Coincidence of the first TNF with relatively large sixth order torque effort was undesirable, in that it would have resulted in system resonance and both large torsional vibrations and high shaft stresses. To ensure that the first TNF did not coincide with the sixth order torque effort, it was necessary first to calculate accurately the "as-found" system's first TNF and then, if necessary, shift the TNF.

In calculating the system's first TNF, careful consideration was given to the motor mass-elastic parameters (Figure 3). It was found that

i) the motor shaft spiders are very stiff
ii) motor shaft spiders are rigidly attached to the laminations as well as to the solid fore and aft discs, and
iii) rotor inertia is distributed over the entire length of the rotor.

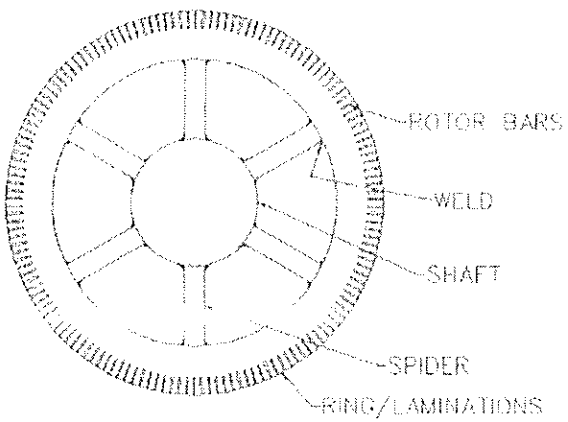

Figure 3, Cross-section of an electric motor rotor.

It is difficult to determine exactly how much the rotor laminations and rotor bars contribute to the overall motor rotor stiffness. Furthermore, distribution of the rotor inertia along the motor shaft has a direct effect on the system TNF. An uncertainty band was quantified (Table 1), to ensure that all possible system mass-elastic scenarios were considered.

Table 1: System TNF Tolerances - Defined

	Uncertainty Band	
	High System TNF	**Low System TNF**
Rotor Spiders/Bars/ Laminations	Significant Stiffening Effect (Zero Length Rotor)	No Stiffening Effect (Rotor Nominal Diameter)
Distributed Rotor Inertia	No Additional Stiffening Effect	Significant Stiffening Effect

The "low system TNF" and "high system TNF" as calculated with the finite-element torsional model were 88.6 Hz and 89.6 Hz respectively. The actual measured first TNF of 90.7 Hz was just outside this defined uncertainty band of 88.6 Hz to 89.6 Hz, so obviously there were additional system mass-elastic uncertainties. Upon considering uncertainties in other system mass-elastic properties such as coupling stiffness (*See Theoretical Discussion p. 6-7*) as well as uncertainty in the measured TNF value, the measured first TNF falls within the calculated TNF uncertainty band.

For all practical purposes the theoretical model, if assessed in terms of a system TNF uncertainty band, does reflect reality. Moreover, given a TNF uncertainty band for the "as-found" system, the TNF's can be confidently repositioned *between* orders of run speed.

For this particular system and all similar systems, the motor mass-elastic characteristics and corresponding uncertainties have a significant effect of the system TNF's and corresponding torsional response characteristics. To design a torsional system by positioning the system TNF's between orders of run speed, a comprehensive system mass-elastic analysis inclusive of uncertainty is necessary.

Case 3 – Data Uncertainty Incorporated Into Torsional Design

The torsional characteristics of motor drive reciprocating compressor systems should be assessed in the preliminary design stage of a project. However, due to logistics the torsional analysis was completed after the design and specification of the following system, comprised of a

- four throw reciprocating compressor operating at 1170 RPM,
- flexible disc coupling, and a
- 2400 HP squirrel cage induction motor.

Analysis of this system considered uncertainty in the system mass-elastic properties. A TNF uncertainty band and "best estimate" were defined (Table 2).

Table 2: System TNF Tolerances – Defined

	Uncertainty Band		
	High System TNF	Best Estimate	Low System TNF
Rotor Spiders/ Bars/Laminations	Significant Stiffening Effect (Zero) Length Rotor)	Significant Stiffening Effect (Zero Length Rotor)	No Stiffening Effect (Rotor Nominal Diameter)
Distributed Rotor Inertia	No Additional Stiffening Effect	No Additional Stiffening Effect	Significant Stiffening Effect
Motor Rotor Inertia	-5%	As Specified	+5%
Coupling Stiffness	+20%	As Specified	-20%
Coup hub Penetration	2/9	2/9	1/3

The proposed system's first TNF uncertainty band was found to be 90.9 Hz – 101.3 Hz (4.7x - 5.2x run speed), as shown in Figure 5. The worst case scenario, fifth order resonance at 97.5 Hz, produced excessively high torsional vibration. The minimum stress design factor at the motor shaft was only 1.1 which was unacceptably low compared to industry standard of 2.0.

Perhaps the most obvious "fix" would have been to directly reduce the shaft stresses by increasing the shaft diameters. Alternatively, the material strength and corresponding stress endurance limit could have been increased. Unfortunately in this instance, it was no feasible to increase the shaft diameters, and higher strength material was uneconomical.

The only practical means of reducing the shaft stress was to reduce the torsional vibration. Torsional vibration was high at fifth order resonance ("worst case" scenario) and could therefore have been minimized by reducing interference of the first TNF with the fifth order torque effort.

Adding an inertia of 4000 lb-in^2 to the compressor shaft center section shifted the first TNF uncertainty band to 84.7 Hz – 94 Hz (4.4x – 4.8x). The shaft stress design factors within the uncertainty band ranged from 1.4 to 1.7. The maximum stress design factor of 1.7 corresponded to the "TNF best estimate" case, and was deemed acceptable by the customer. However, whenever the stress design factor is less than industry standard of 2.0, the predicted TNF's should be verified with field measurements. Moreover, careful attention should be paid to shaft surface finishes and environment factors such as rust.

This case study illustrates the importance of defining a TNF uncertainty band, before modifying a system's mass-elastic characteristics. In this instance the designer wanted to avoid fifth order resonance and also minimize interference of the first TNF with the high fourth order torque effort. Having established the TNF uncertainty band the TNF could be confidently repositioned between the fourth and fifth order of run speed.

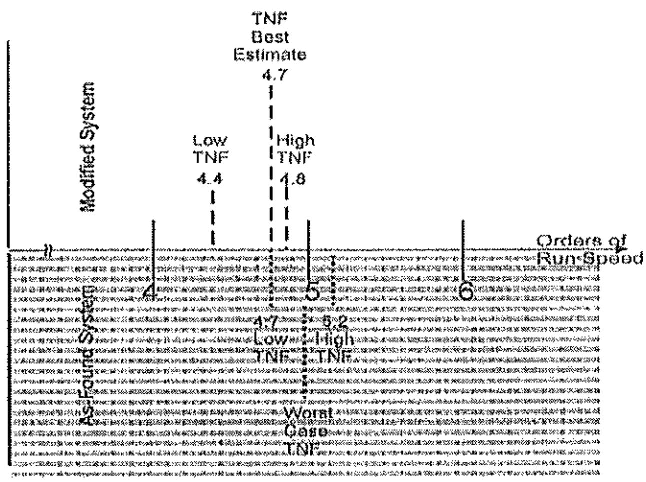

Figure 5, Uncertainty band of modified system is between the 4th and 5th orders.

THEORETICAL DISCUSSION

Torsional Resonance

Torsional forced response is a function of both the compressor torque effort and the dynamic amplification ratio. The compressor torque effort is a function of the compressor operating conditions and load steps. Dynamic amplification is a nonlinear function of the system damping (ζ), forcing frequency (f) and the torsional natural frequency (fn) as defined by the system mass-elastic properties [3].

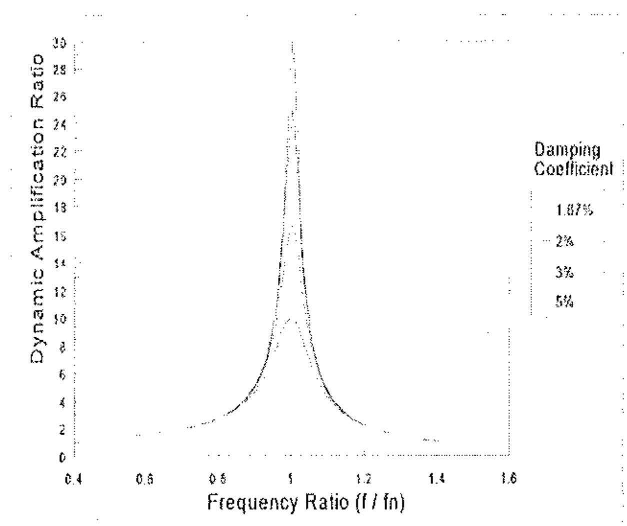

Figure 6, Dynamic amplification at resonance is limited only by damping.

As shown in Figure 6, dynamic amplification is very sensitive to system damping. In fact, at resonance (frequency ratio, f/fn=1) dynamic amplification is limited only by damping. Since the system damping can be very low for a motor driven reciprocating compressor system, the dynamic torsional amplification can be very high. High torsional amplification can result in high shaft stresses so it is good practice to avoid torsional resonance.

Avoiding Torsional Resonance

Although torsional resonance can and should be avoided by re-positioning the TNF, it is not entirely obvious where the TNF *should* be. The compressor torque efforts (i.e. forcing functions) are produced at frequencies corresponding precisely to orders of run speed. Therefore the TNF's should be positioned as far away from the orders of run speed as possible; positioning the TNF's *between* orders of run speed (Figure 7) typically results in an overall minimum torsional amplification of the torque effort. Although the TNF's should be repositioned midway between orders, there are trade-offs between lowering and raising the system TNF's.

The torsional response could be controlled by raising the first TNF to a frequency at which the torque effort is lower. However, it is usually inconvenient or expensive to make a system torsionally stiffer (i.e. higher TNF). Moreover, dynamic amplification is more difficult to minimize at higher frequencies since, as shown in Figure 7, the torsional response curves are wider.

Lowering the first TNF could result in lower dynamic amplification, but the torque effort is generally higher at lower orders of run speed [4]. It is relatively easy to lower the TNF's with a flywheel, but due to the high torque effort at lower orders it is extremely important to precisely position the TNF *between* orders of run speed [5].

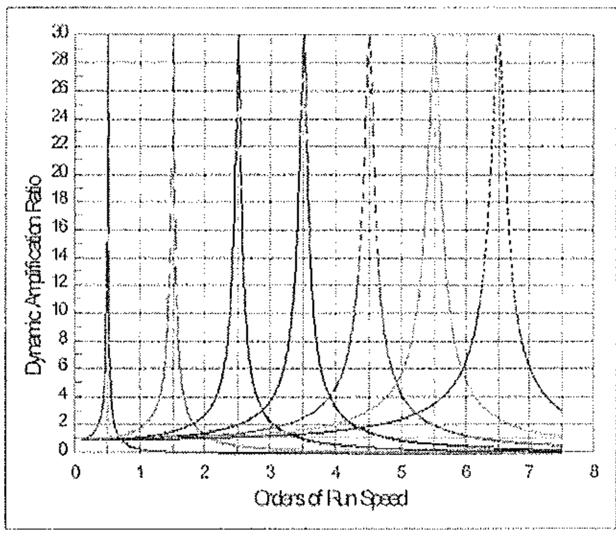

Figure 7, Torsional response curve band width increases with frequency.

Precise TNF Calculation

To ensure that resonance is avoided, precise calculations are imperative. Moreover, all mass-elastic uncertainties must be quantified and a TNF "uncertainty band" established. All the uncertainties tending toward a high TNF are combined to determine the high limit; all those tending toward a low TNF are combined to determine a low limit. Then based on experience, a best estimate within those limits is determined.

Mass-elastic uncertainties associated with the individual system components (Table 3) have been provided by the respective manufacturers Furthermore, uncertainty in 'motor shaft stiffness' and 'coupling hub penetration' are documented [2] and correlated against field measured data. Brief descriptions of the system mass-elastic properties are presented in Table 4.

Table 3, Mass-Elastic Uncertainties

	ID #	Low End of TNF Band	Best Estimate TNF	High End of TNF Band
Motor Rotor Inertia	1	+5%	Spec	-5%
Motor Rotor Stiffness	2	full length shaft	zero length shaft	zero length shaft
Coupling Inertia	3	+1.5%	Spec	-1.5%
Coupling Stiffness	4	-20%	Spec	+20%
Coup Hub Penetration	5	1/3	2/9	2/9
Comp Shaft Inertia	6	+1.5%	Spec	-1.5%
Comp Shaft Stiffness	7	-2%	Spec	+2%
Shaft Stub Location	8	*1" short of full hub*	*Exact Alignment*	*1" beyond hub*

The 'shaft stub location' shown as ID#8 in Table 3, can have a significant influence on the system's mass-elastic characteristics. Ideally the shaft stub should mesh over the full length of the coupling hub; neither extended beyond the hub nor fall short of the full hub length.

Table 4, Description of Mass-Elastic Properties

Motor Rotor Stiffness
Torsional stiffness of a shaft with a circular cross-section is given by:

$$K = J^*G/L$$

where G = modulus of rigidity
J = polar moment of inertia
L = length

The torsional stiffness of a non-circular shaft is not quite as straightforward as that for circular shafts. In particular, the stiffness of rotor shafts (e.g. spiders, laminations, rotor bars) can be very difficult to predict, even if all the physical dimensions are known. For shafts with welded-on spiders the

i) point of rigidity [2] should be applied at the beginning of the rotor core (i.e. the shaft segment which carries the rotor core is assumed to be infinitely stiff), or

ii) rotor inertia should be evenly distributed along the rotor and the shaft stiffness should be increased to account for the spider stiffness.

Motor Rotor Inertia
The motor rotor mass moment of inertia (J) is the summation of all the rotor component inertias. Manufacturing tolerances combine to create a rotor inertia tolerance.

Coupling Inertia
Coupling inertia calculations are based upon the coupling dimensions and the material density.

Coupling Stiffness
Several components including the hubs, spool piece, and flexible discs determine the overall coupling stiffness. The calculated and/or measured stiffness is very sensitive to the grade of material, manufacturing processes, and physical tolerances.

Coupling Hub Penetration
Shafts typically extend several inches into the coupling hubs and as such the torsional load is distributed over the full hub length. This configuration is conveniently represented in terms of coupling hub penetration. A 1/3 hub penetration states that 1/3 of the shaft is free to twist within the hub whereas the remaining 2/3 of the shaft is rigidly attached to the hub.

Compressor Shaft Inertia
The compressor shaft inertia is calculated using shaft dimensions and the material density. An additional rotating inertia proportional to = 2/3 of the connecting rod mass and ½ of the reciprocating mass is applied to each compressor throw.

Compressor Shaft Stiffness
Stiffness of the concentric elements of a compressor shaft are readily calculated from first principles. Theoretical and empirical relations [2] are used to calculate the throw stiffnesses.

<u>Sensitivity of System TNF's to Specific Mass-Elastic Uncertainties</u>

All of the system's mass-elastic uncertainties should be evaluated, but extra consideration should be given to those uncertainties to which the system TNF's are more sensitive.

First of all, consider the mode shape of a typical reciprocating compressor and electric motor system (Figure 8). In this instance the mode shape represents the shaft deflection when the shaft vibrates at the "best estimate" TNF. Generally, the larger the relative deflection, the more significant the mass-elastic uncertainties are at that location.

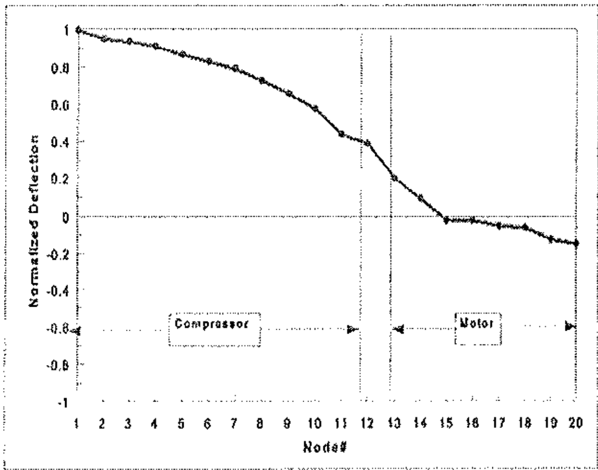

Figure 8, Typical mode shape of a compressor-coupling-motor torsional system.

Secondly, consider the relative influence which each uncertainty has on the first TNF of a typical system, Deviations of the corresponding first TNF's from the system's "best estimate" TNF are shown in Figure 9, for the mass-elastic properties shown in Table 4. The greater the percentage deviation the higher the sensitivity.

Although there is no positive deviation from the "best estimate" TNF for the defined rotor length uncertainty (i.e. it is same as "best estimate" TNF), the negative deviation of approximately 3.75% is relatively large. The first TNF is also very sensitive to coupling stiffness, shaft penetration, and shaft stub location. Motor rotor inertia, coupling inertia, compressor throw inertia, and compressor throw stiffness uncertainties are somewhat less critical. The mass-elastic uncertainties (Table 3) are presented as a percentage of the nominal mass-elastic values. Sensitivity of a system's TNF to specific mass-elastic uncertainties will therefore depend upon the relative size as well as position of the particular component.

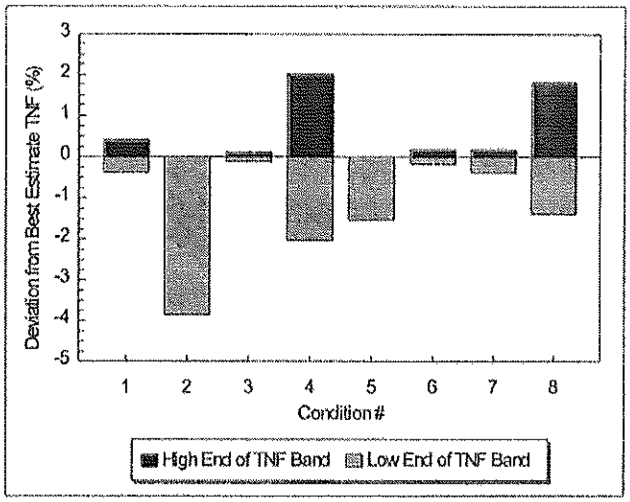

Figure 9, Sensitivity of a system's first TNF to mass-elastic uncertainties.

Large motor driven direct coupled reciprocating compressor systems will all have unique mass-elastic characteristics. Therefore the relative sensitivities shown in Figure 9 should only be used as a general guideline in the torsional design process [1].

A statistical sample of similar motor drive direct coupled reciprocating compressor systems is being compiled by the authors of this paper. This will establish the type of distribution, cumulative effect of all distributions, and the probability density function for the scatter of resonance frequencies.

REALISTIC TORSIONAL MODELS MUST CONSIDER THE INHERENT UNCERTAINTY IN SYSTEM MASS-ELASTIC DATA

The uncertainty in system mass-elastic data is significant and therefore must be incorporated in all torsional analyses. It is reasonable to provide a best estimate of the system TNF, but a tolerance on either side of the best estimate is required. For some systems the TNF will approach the upper limit of the TNF uncertainty band whereas for other systems the TNF will approach the lower limit of the TNF uncertainty band.

Confidence in the torsional model is achieved when the system TNF's are assessed within an uncertainty band. The calculated torsional response, shaft stresses and corresponding stress design factors will then be realistic.

REFERENCES

[1] *A Systems Approach to Torsional Analysis,* Murray, B.D., Howes, B.C., and Zacharias, V. of Beta Machinery Analysis Ltd., Presented at the Gas Machinery Conference, Corpus Christi, Texas, Oct 1995 and the Canadian Machinery Vibration Association General Meeting, Halifax, Nova Scotia, Sept 1995.

[2] British Internal Combustion Engine Research Association (BICERA). *A Handbook of Torsional Vibration.* Compiled by E.J. Nestorides. Cambridge University Press, 1958.

[3] Rao, Singiresu S. *Mechanical Vibrations.* Addison-Wesley Publishing Company, 1986.

[4] *System Torsional Response Over Range of Loading/Operating Conditions,* Murray, B.D., Howes, B.C., and Zacharias, V. of Beta Machinery Analysis Ltd., Nov 1996.

ACKNOWLEDGEMENTS

The authors wish to thank Beta Machinery Analysis Ltd. For its support, and Beta's staff members, especially the Torsional Analysis Group, for their assistance.