A Systems Approach to Torsional Analysis

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ABSTRACT

Case studies (Part I) illustrate problems associated with the torsional design of fixed speed reciprocating compressor systems. In many cases, solutions to these problems demand an iterative systems approach. Part II presents the systems approach, in which the dynamic torsional response of the entire system is determined and the individual components are redesigned until the overall response is acceptable.

Part 1 – Case Studies

1. THE SYSTEMS APPROACH – WHY?

In today's competitive market place productivity is of the utmost importance. The ability to bring a system on-line on time or even early is critical. A delay of a day can result in revenue losses in the thousands of dollars. Accordingly, there is a temptation to build a compressor system based on rules of thumb:

- calculate the mean torque
- calculate "design torque" (mean torque times a service factor based on experience and on the number of compressor throws)
- ensure the coupling and shafts can tolerate the "design torque".

In contrast the systems approach considers the *total effect* of the *interaction* of the components (compressor reciprocating elements, crankshaft, coupling, motor shaft, and motor). Each individual component may be designed appropriately, but the overall effect of the *combination* also needs to be verified. For example, the overall torsional response of a system determines what size coupling is required, but the size and type of coupling must be known before the system's torsional response can be predicted. Similarly the shaft's and coupling's mass-elastic characteristics have a significant influence on motor dynamic response (i.e. electrical current pulsations).

Three cases focusing on fixed speed, electric drive reciprocating compressors are presented. They clearly demonstrate how interaction can generate severe problems and why an iterative approach is needed to solve them.

Trouble-free operation, an important component of productivity, is jeopardized unless a systems approach to torsional analysis is used.

2. CASE 1 – RESONANCE CAUSED EXCESSIVE VIBRATION AT THE COMPRESSOR AUXILIARY DRIVE

Unit: Oil pump driven from the opposite drive end (ODE) of the compressor shaft, with a:

- six throw 3000 HP reciprocating compressor
- squirrel cage induction motor (885 RPM)
- flexible disk coupling.

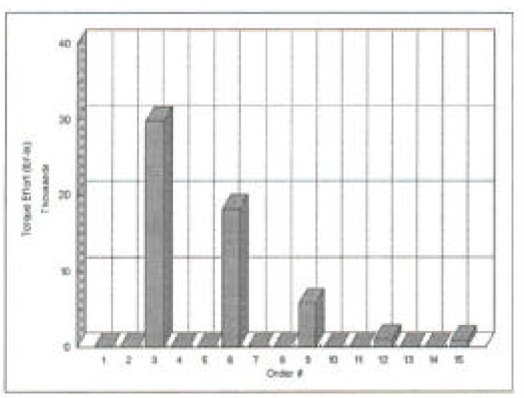

Figure 1. Torque effort was greater at 3X runspeed than at 6X runspeed.

Torsional vibration caused the oil pump drive shaft to fail within one hour of service. The pump was subsequently replaced but again failed within one hour of service.

The compressor torque effort spectrum shown in Figure 1 indicates that the torque effort (input) is primarily concentrated at the 3rd and 6th orders of run speed, with the 3X component much higher.

However, the torsional response curve, Figure 2, is more than 8 times higher at 6X runspeed than at 3X. This difference suggests resonance, and leads to the conclusion that the torsional natural frequency (TNF) is close to 6X run speed (88.5 Hz).

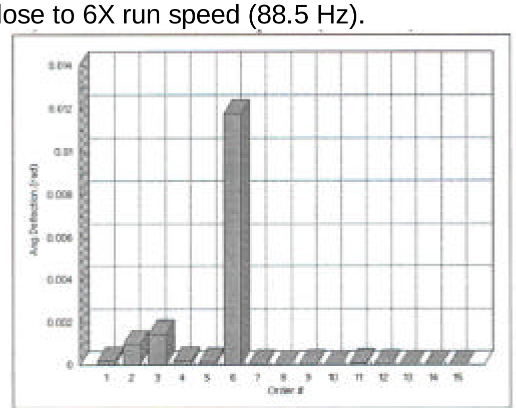

Figure 2. Torsional response at compressor ODE was excessive at 6X runspeed.

Based on measured data and a finite element torsional model, the system's TNF was determined to be approximately 87 Hz (5.9X run speed), too close to the known high input torque at 6X runspeed. To make matters worse, the pump's drive shaft had a TNF of 93 Hz, or 6.3X runspeed. The combination of all these factors was intolerable.

To reduce the dynamic 6th order amplification, coincidence of the TNF with the 6th order torque effort had to avoided. We added three small donuts (each 4630 lb.in²) to the compressor shaft, as shown in Figure 3. This modification effectively lowered the system's TNF from 87 Hz to 79 Hz which, as shown in Figure 4, reduced the predicted overall angular displacement at the compressor auxiliary drive to 23% of that in the existing system.

At the time of writing, the system has run continuously for several weeks with no damage to the pump.

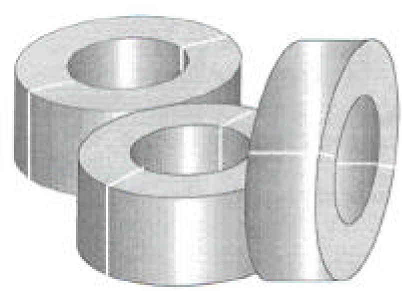

Figure 3. Three donuts added to the compressor shaft solved the problem.

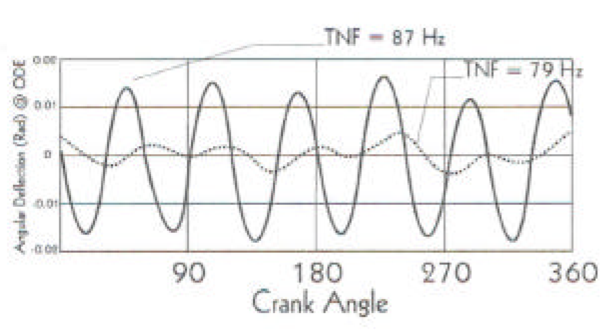

Figure 4. Moving the TNF to 79 Hz from 87 Hz reduced the amplitude significantly.

3. CASE 2 – COUPLING TOO STIFF FOR PROPOSED MOTOR/COMPRESSOR SET

The proposed system consisted of a:

- four throw compressor
- 1250 HP 1175 RPM squirrel cage electric induction motor, with a shaft designed for a different service
- flexible disk coupling.

Two modifications are discussed:

- addition of a small flywheel (20,000 lb.in²)
- introduction of a "torsionally soft" coupling.

Stresses were evaluated against a design factor, which should be greater than 2. See Part II, Section 4. As shown in Figure 5, only the system with the "torsionally soft" coupling met the stress criterion.

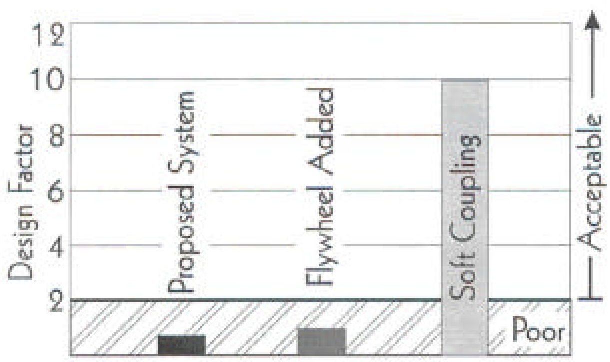

Figure 5. A design factor of less than 2 is unacceptable.

The calculated torsional natural frequency (TNF) of 97 Hz was close to 5X run speed. As shown in Figure 6, there was significant deflection, resulting from interference of the TNF and the torque effort at 5X runspeed.

The flywheel modification reduced the system TNF to about 89 Hz (4.5X runspeed), but as shown in Figure 7, peak deflection is still obvious at 3, 4, & 5X runspeed. These components contributed to the excessive stress indicated in Figure 5.

The "torsionally soft" coupling modification dropped the system TNF to about 10 Hz (0.5X runspeed). As shown in Figure 8, the peak deflection was then minimal.

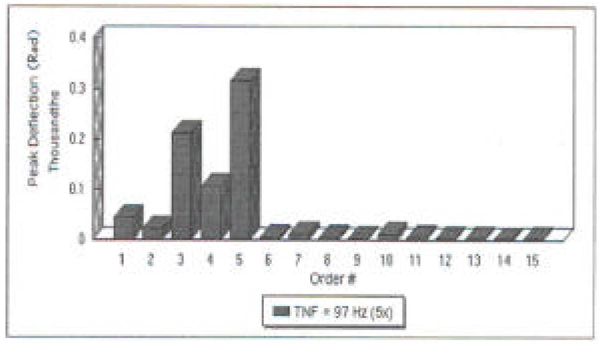

Figure 6. Original system had noticeable deflection at 1 through 5X runspeed.

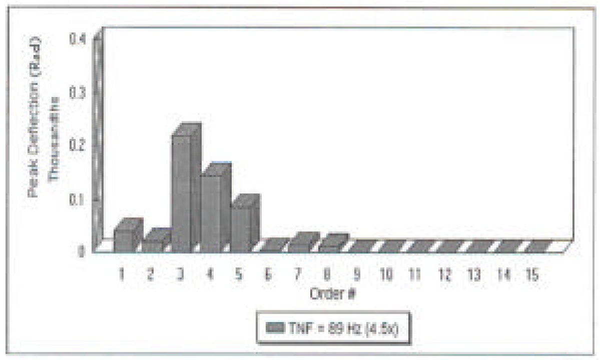

Figure 7. Flywheel reduced deflection at 5X but increased it at 4X runspeed.

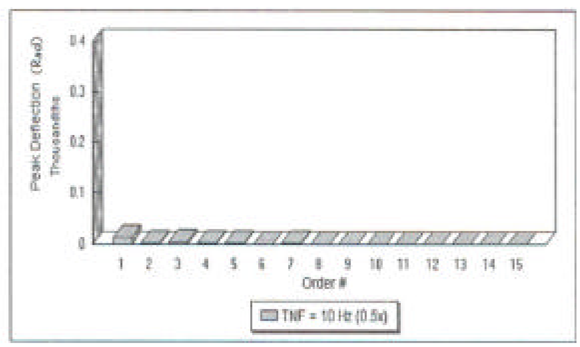

Figure 8. Torsionally soft coupling reduces deflection to almost nil.

4. CASE 3 – ABSENCE OF FLYWHEEL ALLOWED EXCESSIVE CURRENT FLUCTUATION

The proposed system consisted of a:
- four throw compressor
- 1375 HP induction motor, with a rated speed of 992 RPM at full load,
- flexible disk coupling.

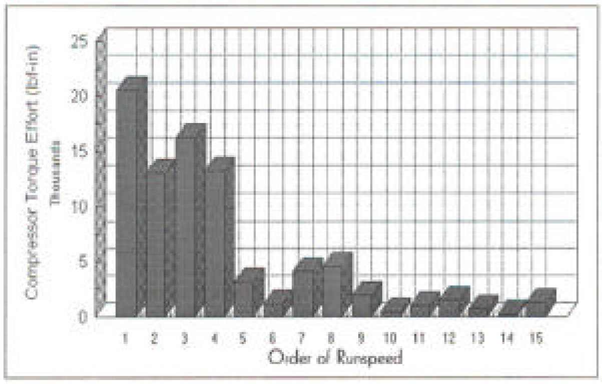

Figure 9. Significant torque is apparent at 1, 2, 3, & 4X runspeed.

The system torsional natural frequency (TNF) interfered with 5X runspeed, but the torque effort at that frequency was small, as shown in Figure 9. Damping controlled the torsional response and therefore shaft stress was found to be acceptable.

However, motor dynamic response was found to be unacceptable. In the proposed configuration, motor speed fluctuated from 981 to 998 RPM. The predicted electrical current pulsation from this speed fluctuation was 80%, which exceeds the NEMA standard of 66%, and which is double the new API standard of 40%.

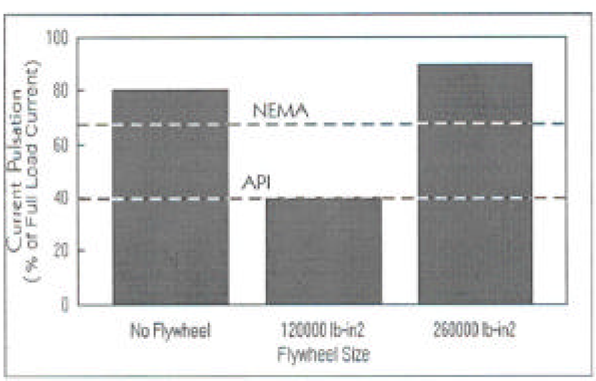

Figure 10. A precisely sized flywheel effectively controlled electrical current pulsation.

The solution to the excessive current fluctuation was to add a flywheel to the compressor shaft. As shown in Figure 10, a large flywheel (260,000 lb.in²) made the problem worse, the system TNF coincided with the 3rd order torque effort.

A smaller flywheel (120,000 lb.in²) decreased the torsional response, and thereby reduced the current fluctuation to an acceptable value of 40%.

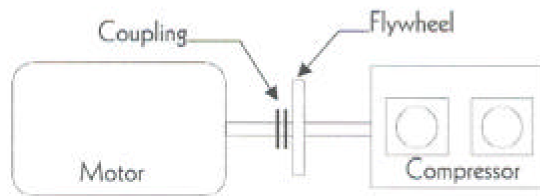

Figure 11. The new flywheel was added close to the coupling on the compressor side.

PART II – System Torsional Analysis Procedures

The torsional analysis of a fixed speed reciprocating system is a seven step procedure:
- determine the system's torsional natural frequencies
- determine the compressor torque effort
- predict the amount of interference between torque effort and torsional natural frequency (TNF)
- evaluate the torsional forced response, including damping
- modify the system until stress is acceptable
- evaluate electrical current pulsations, and remodify if necessary
- confirm results over the range of operating conditions.

1. DETERMINE THE SYSTEM'S TORSIONAL NATURAL FREQUENCY (TNF)

TNF is a ratio of rotational stiffness to rotational inertia. Comparing a torsional system to a simple spring-mass system, the following are analogous:
- inertia (Jo) to mass (m),
- rotational stiffness (K) to linear stiffness (K_l), and
- angular deflection (T) to translational deflection (x).

Inertia (Jo) is essentially a function of density and geometry, and is determined for the system by adding the inertia for each element.

$$Jo_{system} = Jo_{motor} + Jo_{coupling} + Jo_{compressor} \tag{1}$$

System rotational stiffness is determined from Equation (2), where K is a function of component material, diameter, length and in some configurations, loading.

$$\frac{1}{K_{system}} = \frac{1}{K_{motor}} + \frac{1}{K_{coupling}} + \frac{1}{K_{comp}}. \tag{2}$$

Several natural frequencies are usually present in the system, and it is important to identify each of them.

2. DETERMINE THE COMPRESSOR TORQUE EFFORT

The torque effort (input torque) acting on the shaft in any reciprocating compressor installation is created by a combination of cylinder gas forces and reciprocating inertial forces. These forces are both functions of crank angle but are not necessarily in phase with one another. As shown in Figure 12 for a loaded cylinder, the torque effort curve of each cylinder is periodic but is not sinusoidal; it is comprised of several component frequencies.

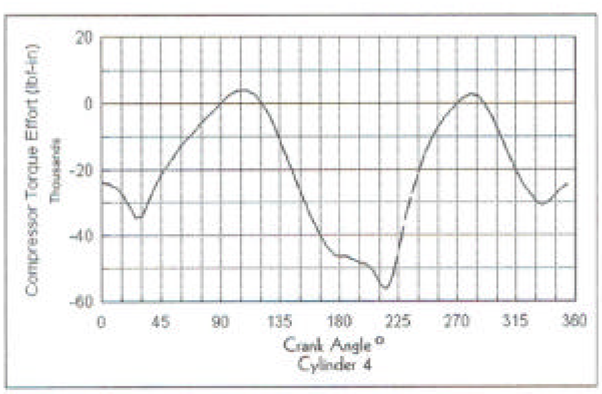

Figure 12. The individual cylinder torque effort curve is not sinusoidal.

To further complicate the analysis, the compressor torque effort is the combination of all the individual cylinder torque effort curves and their respective phasing. It is this combination curve that must be considered in the analysis. Figure 13 shows the overall torque effort curve for a four cylinder unit.

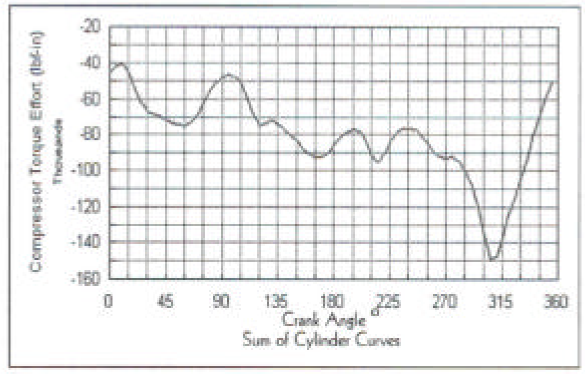

Figure 13. The system torque effort curve is a combination of the cylinder curves.

For the purpose of analysis, it is necessary to determine the frequency content of the torque effort curve; that is, to Fourier transform the complex time domain torque effort curve into the frequency domain, as shown in Figure 14.

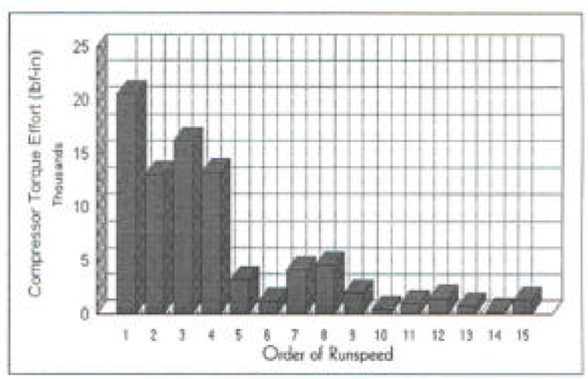

Figure 14. The FFT of the system torque effort shows relative torque at each frequency.

Since each compressor throw experiences one complete torque fluctuation every revolution, the torque amplitude at the first harmonic would likely be large as compared to the other harmonics. However, large amplitudes may also be seen at the higher harmonics.

Note that the compressor torque effort acts at several locations along the shaft. The length of the shaft between the cylinder throws has a *significant* effect on the system's torsional response, and it is therefore incorporated into the torsional model.

3. PREDICT INTERFERENCE BETWEEN TORQUE EFFORT AND TNF

A key component of the analysis is the interference of torque excitation with torsional natural frequencies. Where the TNF's and the torque effort frequencies are separated sufficiently, response is likely to be small, and the system is likely to operate well. Where the frequencies coincide, the phenomenon of ***resonance*** results, and the system may have severe problems. See Figure 15.

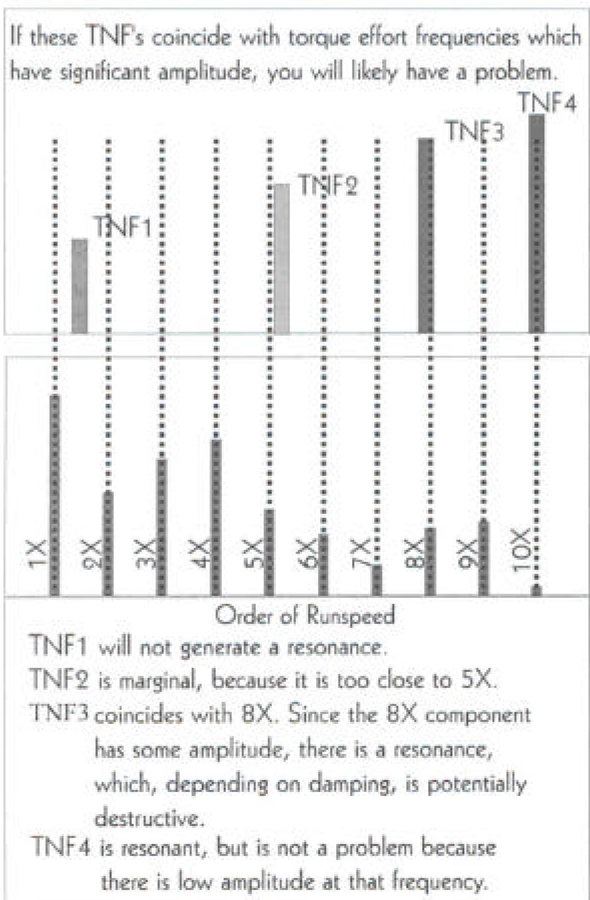

Figure 15. Angular deflection at resonance is limited only by damping.

4. EVALUATE THE TORSIONAL FORCED RESPONSE, INCLUDING DAMPING

Torsional forced response depends on both interference (of the torque excitation with system torsional natural frequencies) and damping. A reasonable but conservative amount of damping is introduced into a finite element model, and the resulting forced torsional response is predicted.

As shown in equation (3) and illustrated in Figure 16, torsional forced response (dynamic amplification) is a non-linear function of interference (r) and damping (?).

At resonance, r = 1, and the angular deflection is limited only by damping, as shown by equation (4).

$$\frac{T}{T_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2z\,r)^2}} \qquad (3)$$

where
T = angular deflection
T st = static deflection
. = damping
r = $\frac{w \bullet Order}{w_n}$
w = runspeed in radians per second
w_n = torsional natural frequency

$$\frac{T}{T_{st}} = \frac{1}{2\,.} \qquad (4)$$

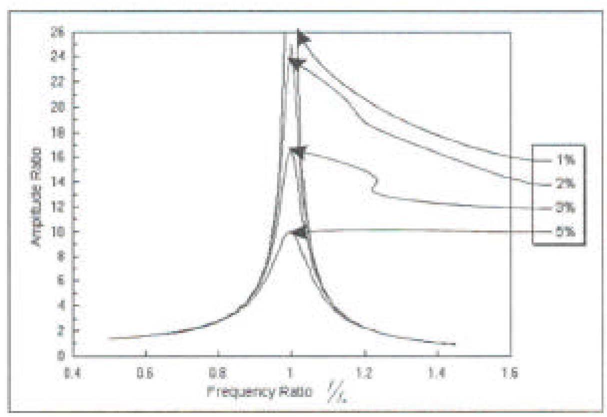

Figure 16. Angular deflection at resonance is limited only by damping.

Shaft stress is calculated from overall peak deflection, which in turn is a vectorial addition of components in the peak deflection spectrum. The maximum value of any one peak is not useful as a limit, because it does not provide adequate protection when there is energy at several frequencies.

Stresses are evaluated against a design factor which incorporates calculated mean and dynamic stress, material endurance limit, and ultimate strength. The design factor should be greater than 2.

5. MODIFY THE SYSTEM UNTIL STRESS IS ACCEPTABLE

The torsional analysis procedure is an iterative one. If stress levels throughout the system are not acceptable, the system must be modified and the analysis repeated.

6. EVALUATE CURRENT PULSATIONS

At this point, shaft stresses are acceptable, but what is the effect on the motor? The torsional fluctuations we have described cause the motor speed to fluctuate about the mean operating speed. These speed fluctuations cause the motor *load* to fluctuate, and that in turn causes pulsations in the electrical current demand. Therefore, another reason to limit torsional response is to ensure that the system can meet electrical current fluctuation guidelines.

Once a design that meets the torsional stress criteria has been determined, speed fluctuations of the motor are calculated. If the selected guideline (NEMA = 66%; API = 40%) is not met, the process is repeated.

7. CONFIRM RESULTS OVER THE RANGE OF OPERATING CONDITIONS

A complicating factor in doing any compressor analysis is that response can vary radically with operating conditions. In particular, a “rubber compressor”, one with an extremely wide range of operating conditions, poses additional problems. Only when all the criteria are met for all operating conditions is the job done.

8. THE SYSTEMS APPROACH TO TORSIONAL DESIGN, CONFIRMED

The case studies presented here demonstrate the need for a systems approach. In each case, it was the interaction of the components that caused the problem.

It is essential to look at the overall system. Unless all factors are taken into account, the individual components can easily be either inadequate or overdesigned. Although the systems approach initially takes more time than the rules of thumb approach, in the long run, the systems approach is more economical.

9. REFERENCES

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10. ACKNOWLEDGMENTS

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